INSTABILITIES IN FERROFLUIDS WITH TEMPERATURE AND MAGNETIC FIELD DEPENDENT VISCOSITY UNDER DIFFERENT MODULATIONS

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ABSTRACT

The linear and non-linear stability of a ferrofluid with temperature and magnetic-field-dependent viscosity subjected to an imposed time-periodic gravity modulation, rotation modulation, boundary temperature modulation and magnetic field modulation is studied. The perturbations in the system due to external reasons are expanded in terms of the amplitude of modulation in the case of linear stability and truncated representation of the Fourier series in the case of non-linear stability. A non-autonomous Lorenz model for the problem is first obtained and both linear and non-linear analyses of the system is performed using it in the case od gravity, rotation and magnetic field modulations. The expression for the critical Rayleigh number R_0 and the correction Rayleigh number R_{2c} is found from the linearized Lorenz model. However for time-dependent boundary temperature modulation we follow the Venezian approach for the linear stability nalysis. The Lorenz system of equations is solved for the amplitude to arrive at the Nusselt number (Nu) which quantifies the heat transport for all the modulations. In the study we find that the thermorheological effect and the magnetization effect work in unison to destabilize the system while the magnetorheological effect stabilizes the system. The influence of the parameters on the heat transport is opposite to their effect on the critical Rayleigh number. Further, the square waveform is found to facilitate maximum heat transfer for all the modulations. Results of the study agree quite well with those of limiting cases.

Keywords:

Ferrofluid, Variable Viscosity, Rheological Parameters, Magnetic Numbers, Modulation, Waveforms.

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Chapter 1

Introduction

Fluid mechanics which studies flow as the crux of life keeps us closely related with various phenomena of nature like the flow of air, water etc., the movement of social information which plays a vital role in real life aspects. The principles of fluid mechanics over the last century has immensely contributed to the augmenting technologies ranging from various industrial to engineering applications. The last century has witnessed the meticulous understanding of the laws and principles of fluid mechanics and the skills involved in encountering diverse challenging problems of the real world. Fluid mechanics has innumerable applications marked over a broad scale of disciplines of science and engineering ranging from aeronautical, chemical engineering, bio-medical, marine, civil to geophysics.

1.1 Rayleigh-Bénard Convection (RBC)

The variation of densities between two parallel plates caused by temperature difference in a horizontal fluid layer gives rise to convection which has remarkable significance owing to its applications in science and engineering. Such convection in literature is known as Rayleigh-Bénard convection (RBC). The experiments conducted by Bénard Henri [9] gave rise to the RBC problem. Based on the exploratory studies by Lord Rayleigh [66] convection occurs only when the destabilizing action of the temperature difference is strong enough to overcome the frictional action of viscosity and thermal conductivity.

Rayleigh Number

The temperature difference at which the motion occurs in the RBC is measured by a non-dimensional number called Rayleigh Number (R) given as

$$R = \frac{\alpha g \rho_0 \Delta T d^3}{\gamma \kappa} \tag{1.1.1}$$

According to the studies made by researchers on the archetypal complex flows we observe that convective transport of heat and mass play a vital role in numerous phenomena of nature. Thus fluid motion accords the existence of life in general and all this clearly indicates the steadfast interest of researchers in convection. With RBC as the origin of the problem we proceed to present a brief introduction of the various aspects of this study.

1.2 Ferrofluids

Ferrofluids also known as magnetic fluids are stable suspensions of colloidal ferro or ferri magnetic particles of the order of 10 nm and are colloidally dispersed in a magnetically passive liquid [11, 16, 57, 69, 72]. These colloidal particles are coated with monomolecular layer of surfactants to avoid their agglomeration. Ferrolfluids become strongly magnetized in the presence of a magnetic field i.e. these fluids under the influence of an external magnetic field, exhibit a large magnetization and as soon as the field is removed, the fluid attains its zero magnetization state at once. Further, an external magnetic field can readily restraint the advective transport in a ferrofluid and hence they find innumerable applications in problems of heat transport. With a significant growth of microscale heat exchangers in micro-electromechanical system (MEMS) and nano-electromechanical system (NEMS) gadgets, ferrofluids have become a prime component to understand the heat transports (enhancing or delaying convection) in fluid systems with the need to control convection in a non-isothermal situation. The inherent pliability of ferrofluids opens up a whole range of applications in science and technology ranging from instrumentation, vaccum technology, electrical and electronics engineering, biomedical applications and many more. Detailed works by Rosenweig [71] and Odenbach [56] provide an extensive literature on ferrofluids and their applications. The magnetic field, temperature and the density of a ferrofluid have a huge impact on its magnetization such that a small difference in these quantities leads to a change in body force distribution in the fluid giving rise to convection in ferrofluids in the presence of magnetic field gradient, known as ferroconvection.

Some of the biomedical applications of ferrofluids are magnetic drug targeting, hyperthermia, contrast enhancement for magnetic resonance imaging (MRI), magnetic separation of cells, X-ray analysis, cancer detection and for positioning tamponade for retinal detachment restoration in eye

surgery. In engineering applications ferrofluid is widely used in sealing of the hard disk drives, rotating X-ray tubes , rotating shafts. However in the field of electronics ferrofluids find one of its major application in controlling of heat in loudspeakers.

1.3 Variable Properties

Most studies of the problems of heat transport are based on the constant physical properties of the ambient fluid. However, it is known that these properties may change with temperature, especially for fluid viscosity. To accurately predict the flow and heat transport rates, it is necessary to take account of this variation of viscosity. This distinctive thermophysical property of a ferrofluid has captured the attention of researchers to study their influence on various parameters governing the system. One such being the viscosity of the ferrofluid finds its utility in innumerable applications of technology, nature and engineering problems of convective flow. In these phenomena viscosity is found to vary with temperature which is one of the significant thermo-physical property to study convective instabilities. Temperature and magnetic field are known to have a great influence on the viscosity of the fluid and these are respectively known as thermorheological and magnetorheological effects. Thus the consideration of a varying viscosity which helps in accurately predicting the flow and heat transport rates serves as a motivation to incorporate a temperature and magnetic field dependent viscosity to study ferroconvection. In this study the following two cases of the effect of variable viscosity parameter, V are considered:

(i) $V < 0$, dominance of magnetic field dependence of viscosity over temperature dependence - magnetorheological effect,

(ii) $V > 0$, dominance of temperature dependence of viscosity over magnetic field dependence - thermorheological effect,

1.4 Modulations

In several studies on the effect of modulation, it has been shown that by applying proper transition to a controlling parameter, it is possible to stabilize or destabilize the conduction state. Hence, this concept of applying transitions to a controlling parameter is very important due to its utility as an effective tool to control the onset of convection. Modulations have proven to be very effective in different scientific fields. In computer graphics, transforming 2D images into the frequency domain

and applying the inverse transform after modulation is a common practice in image processing. We proceed to understand the following four types of modulations considered in the study, viz.,

1. **Gravity Modulation (g-jitter)**

   A physically important class of problems involves convection in a fluid layer in the presence of time dependent gravity field. The buoyancy force resulting from the interaction of the density gradient and gravitational field shows a complex spatio-temporal structure. The time-dependent gravitational field is of interest in space laboratory experiments, in areas of crystal growth, large scale convection in atmosphere and other applications. This thermo-gravitational vibration (called gravity modulation or g-jitter) is known to be an effective means of controlling instabilities. Existence of adverse density variations within the fluid and a body force are the necessary conditions to initiate natural convection. In general, a liquid layer with density gradient that is convectively stable under constant gravity condition can be destabilized if a time-dependent component of the gravity field is introduced. Certain combinations of thermal gradients, physical properties and modulation parameters may lead to parametric resonance and hence to the instability of the system. Thus this idea of using mechanical vibration as a tool to improve the heat transport rate has gained considerable attention in recent decades due to its important applications in engineering and technology.

2. **Rotation Modulation**

   We now emphasize on the effect of another important external constraint which is a consequence of inertia (coriolis force) know as rotation modulation. Coriolis force finds its applications in the fields of long-range gunnery, oceanography, geophysical and astrophysical flows among others. When we study the rotation effect, the stability of the flow is affected by one or more parameters in the form of rotation speed. The inhibiting effect of rotation on the instability of the fluid layer heated from below has long been recognized as a phenomenon of major importance in RBC. The convection in rotating systems is relevant to many geophysical applications. Coriolis force due to rotation has been extensively used to suppress RBC in a horizontal layer of fluid heated from below. The vertical shear generated by rotation is responsible in suppressing the convection by balancing a part of potential energy produced by an unstable nature of buoyancy force. In this study, we aim to consider the time dependent coriolis force to study ferroconvection with variable viscosity.

3. **Boundary Temperature/Thermal Modulation**

   Most of the studies on ferroconvection are focused on the linear analysis of the system with a

steady state temperature gradient. However the growing importance of a hydrodynamic system in various experimental and theoretical setups has drawn the attention of many researchers for a thorough study. Further we observe that an external constraint where the temperature gradient is explicitly dependent on both space and time has caught the attention of researchers owing to the innumerable applications of practical significance. In various pragmatic scenarios the transitory heating or cooling at the boundaries gives rise to non-uniform temperature gradients which are space and time dependent. This problem which presumes the energy equation as its solution under appropriateu time-dependent boundary conditions is known as the temperature/thermal modulation problem. These profiles can be used as an effective mechanism to control the convective flow by suitable tuning of the parameters like, amplitude and frequency of modulation. The present work deals with the study of the effect of time periodically varying boundary temperatures (thermal modulation) for three different cases viz.,

(i) Synchronous Modulation (Inphase, $\Phi = 0$)

(ii) Lower Wall Modulation ($\Phi \to -i\infty$)

(iii) Asynchronous Modulation (Out of Phase, $\Phi = \pi$)

4. **Magnetic Modulation**

Convective instability in a system is well controlled by various externally imposed mechanisms which serve as an effective tool to control the onset of convection. One such is by applying the externally imposed vertical magnetic field. We see that thermal convection is affected by magnetic fields in the outer layers of the sun and other late type stars. Also, various phenomena of astrophysics are a consequence of magneto-convection occurring in various depths of convection zone. This deals with the interaction between convection and an externally imposed magnetic field in a Boussinesq fluid. Thus, the heat transport in the system can be efficiently controlled with the help of magneto convection. The strength of magneto convection is measured by Chandrasekhar number,Q, which is the ratio of the Lorentz force to the viscous force. The Lorentz force is exerted by the magnetic field, if this force was weaker than viscous or turbulent pressure then the convective motions twist and stretch the magnetic field, which in a turbulent flow increases its strength. If the Lorentz forces are stronger than the viscous forces or turbulent pressure forces, then the magnetic field channels the plasma motions along the field direction and inhibits the convection. If one wants to get control on magneto convection it is necessary to find an effective magnetic time dependent Rayleigh number. This is where the idea of the time dependent magnetic field (magnetic field modulation) occurs. Further, the

synergy between an electrically conducting fluid flow and the applied magnetic field emanate magneto-convection. The dependence of temperature of the fluids on magnetization also results in convection. This feature makes them advantageous in space research, where a magnetic body force can replace the role of gravity. This magnetic field modulation in a RBC problem which can easily regulate convective motions is an essential part of this study owing to their innumerable applications in the field of oceanography, astrophysics, space research among others.

1.5 Waveforms

Non-sinusoidal waveforms contain multiple sinusoidal waves (trignometric sine or cosine) of different frequencies in contrast to a sine wave which consists of a single frequency. Ruckmongathan [73] showed that besides the multisteps, other waveforms like trapezoidal and triangular exist in electronics for reducing power dissipation in liquid crystal displays. Most of the electronic components and systems continue reduction in size while growing in energy density resulting in the need for more efficient thermal management and in such situations different waveforms can be effectively used. This is well comprehended from the theoretical studies of non-sinusoidal waveform like Square Wave (SQW), Triangular Wave (TW) and Sawtooth Wave (STW) which play a vital role in several electronic components and other industrial processes. In the studies of Zhao et al. [121] we see that that there is an existence of other waveforms in oscillating heat pipes (OHP) such as the triangular waveform, in addition to the sinusoidal waveform (SW). Also, in the work of Yin and Ma [120], an oscillating motion with a triangular waveform shows that triangular waveform gives a higher heat transport coefficient in comparison to the sinusoidal waveform. The recent works of Siddheshwar and Kanchana [98] and Siddheshwar and Meenakshi [100], concern three waveforms ,viz, trigonometric sine, triangular and square for vertical oscillations but their work involves constant viscosity non-magnetic fluid. However, very few attempts have been made to study the effect of heat transport in the context of square and saw-tooth waveforms. Thus, the non-sinusoidal waveforms modeled by an infinite Fourier series makes a significant contribution in understanding the heat transport in a fluid. This serves as a motivation for us to study the waveforms. Thus in this study we focus on studying the modulations using four different waveforms, viz, SQW, SW, TW and STW.

Chapter 2

Review of Literature

One of the extensively studied hydrodynamic pattern is the RBC [17], where we consider a horizontal layer of fluid that is heated from below and cooled from above. In this study the RBC is studied for a ferrofluid with variable viscosity which is externally modulated by varying gravity, rotation, temperature and the magnetic field for sinusoidal and non-sinusoidal waveforms. In the section to follow we present the literature for each of these.

2.1 Rayleigh-Bénard Ferroconvection (RBF)

We present a few works which have studied ferroconvection starting from the ground breaking work of Finlayson [25] which analysed the stability of a ferrofluid in the presence of a uniform vertical magnetic field followed by the experimental works of Schwab et al. [77] where they observed how the critical Rayleigh number was impacted by the magnetic field during the onset of convection. Then the work of Odenbach [55] studied ferroconvection in a hollow cylinder under microgravity, followed by the works which provided a detailed understanding of ferrofluids [56, 58]. Pertaining to the significance of ferrofluids, studies by various authors viz., Gupta and Gupta [35], Gotoh and Yamada [33], Stiles and Kagan [110], Kaloni and Lou [41], Maruthamanikandan [48] and Siddheshwar [88, 94] have contributed to the comprehension of critical aspects of ferroconvection.

2.2 Gravity Modulation in RBC and RBF

The work by Gresho and Sani [34] and Gershuni and Zhukhovitskii [31] throws light on how in a problem of RBC, vertical oscillations can bring in stability in the system. Their work which was the first study on the effect of gravity modulation in a fluid layer shows that stability can greatly be influenced at appropriate frequencies and amplitudes and this results in synchronous or sub-harmonic

modes of instability. The work by Wadih et al. [118, 119] considers the effect of gravity modulation on heat transport in Newtonian liquids. Siddheshwar [93] reports the first work on the gravity modulation problem using the Ginzburg-Landau model. However, the effect of gravity modulation on ferroconvection in the constant viscosity case was investigated by Bajaj [8, 7].

2.3 Rotation Modulation in RBC and RBF

The elementary work on effect of rotation in a convection phenomena was done by Chandrasekhar [20] where he studied the theory of a fluid heated from below subjected to a coriolis force. Later on Veronis [117] studied the steady motion in a fluid under the effect of rotation with finite amplitude. Then Bhattacharjee [15] threw light on the effect of rotation modulation in the RBC. With this ground work, authors like [116], [113],[5],[1],[51], [85], among others have studied the effect of rotation modulation in ferrofluids and contributed to some peculiar results. One such recent work by Siddheshwar et al. [101] concerns the theoritical understanding on the onset of RBC in a horizontal ferrofluid layer subjected to rotation with respect to the vertical axis, however they consider constant viscosity case with no effect of waveforms which is an important objective of the present work. Further, the most recent work of Kanchana et al. [42] studies the effect of rotation modulation on RBC in constant viscosity non-magnetic liquid.

2.4 Temperature Modulation in RBC and RBF

The convective flow in a fluid system is known to be best controlled by modulations in temperature as seen in the detailed review by Davis [22]. We emphasize on the pioneer work of Venezian [115] where he studied the effect of thermal modulation on the temperature instability in a horizontal fluid layer as a thermal analogue of the experiments of Donnely [23]. This pathfinder work resulted in a number of works by other researchers which study various aspects of onset of instability, heat transport and effect of modulation on various non-magnetic fluids [31, 67, 68, 26, 102, 105, 95, 42, 109]. However the effect of thermal modulation in ferrconvection is studied by [106, 107] where they discuss the onset of instability of temperature modulated thermomagnetic convection by using the Floquent theory. An investigation by Singh et al. [108] shows an interesting global stability result which shows that the onset of instability is favoured by a low frequency modulation while for high frequency modulation there is no effect.

2.5 Magnetic Modulation in RBC and RBF

Thompson [114] and Chandrasekhar [19] were the first to study magnetoconvection in a horizontal fluid layer and they found that vertical magnetic field delays the onset of convection. By using the Galerkin method, Nakagawav [49] and Jirlow [39] showed that a vertical magnetic field suppresses the onset of convection. Finlayson [25] considered a magnetic horizontal fluid layer heated from below in the presence of a uniform vertical magnetic field. He performed linear stability analysis by considering free–free and rigid–rigid boundaries, and predicted the critical gradient of temperature corresponding to the onset of convection, considering both buoyancy and magnetic forces. Rudraiah and Sekhar [75] treated Finlaysons problem with an internal heat source and showed that the variation of temperature, due to the heat source, induces a variation in the magnetic field. These variations can be used to control magnetic convection. All of above studies were done considering a uniform external magnetic field. Motivated by the applications several authors investigated the suppression of convection by strong magnetic field. Siddheshwar and Pranesh [102, 104], Bhadauria [12], Siddeshwar et al. [95] have investigated the effect of temperature / gravity modulation on the onset of convection with uniform magnetic field using linear /non-linear analysis. The onset of horizontal RBC in ferrofluids, and its dependence on magnetic field gradient is investigated by Aniss et al. [4]. Aniss et al. [3] have investigated the problem of convection using time dependent magnetic field (magnetic field modulation). Their studies are restricted to static and linear linear laws of magnetization. Kiran et al. [43] have studied oscillatory mode of nonlinear magneto-convection under time dependant magnetic field in a non-magnetic fluid. the recent work of Kopp et al. [44] investigated the dynamics of unsteady heat transport for various types of modulation of external fields and for different profiles of the angular velocity of the rotation of electrically conductive fluid. However, the literature shows the incorporation of the time dependent magnetic field mostly for non-magnetic fluids with constant viscosity. Thus, in this study we aim to incorporate the effect of magnetic modulation for ferroconvection with variable viscosity in both the linear and non-linear cases.

2.6 RBC and RBF with Variable Viscosity

In this section we emphasize on the literature which studies variable viscosity in magnetic and non-magnetic fluids. The critical Rayleigh number for a liquid was found to decrease with a small cosine variation of viscosity in the works of Palm [59] and Jenssen [38] by using a perturbation method. Hoard et al.[36] gave the direct quantification of the critical Rayleigh number in liquids with drastic variations in viscosity. Using the asymptotic approach, Severin and Herwig [82] investigated the

outcome of variable viscosity on the onset of instability in the RBC problem. A significant study by Straughan [111] showed that nonlinear stability boundaries are sharp when compared with the instability thresholds of linear theory. A numerical study on liquids with weak electrical conductivity was performed by Siddheshwar [92] to study the thermorheological influence on magnetoconvection. The few works by Maruthamanikandan [48], Siddheshwar and Maruthamanikandan [99], Sunil et al. [1], Nanjundappa et al. [50], Sekhar and Jayalatha [78, 79], Siddheshwar et al. [103] and Sekhar et al. [80] consider either thermorheological or magnetorheological effect on ferroconvection. However, Sekhar et al.[81] studied the problem of ferroconvection with both rheological effects using the Galerkin and the shooting method. Another study by PGS et al. [97] studies the effects of variable viscosity and thermal modulation in a Rayleigh-Bénard electro-convection for a non-magnetic fluid. But both these studies are restricted to a linear stability analysis.

Chapter 3

Need for the study

The broad spectrum of applications of buoyancy driven convection of Newtonian fluids like ferrofluids provides the background and motivation for the problem to be studied in this work. In the recent years there has been an exponential progress in the wealth of experimental data and theoretical studies on ferrofluids as seen in the literature. Also, there lies a huge necessity to understand the complexity of ferrofluid theory as this emerging area of application of ferrofluids paves way for advanced research accompanied by broad range of ideas, challenges and applications. Studies related to heat transport characteristics of ferrofluids not only present the challenges of solving theoretical problems but also finds growing importance for technical and medical applications along with the wide range of captivating questions in physics such as the influence of dipolar interactions and magnetic fields on the structure and dynamics of fluids. This helps to address the phase transitions in such systems efficiently. The effect of an applied external magnetic field on the physical properties of the fluid also paves way for several technical and medical applications [76, 18, 47, 40]. Further, the effect of modulations on the onset of thermal instability has become one of the most interesting research area in the recent past as in various practical scenarios it is possible to delay or advance the onset of convection. Thus this problem is of relevance and interest in innumerable ways.

It is worthy to note that the literature on ferroconvection is extensive in the case of linear stability analyses and sparse in so far as non-linear stability is concerned. Further, the literature review so far mainly concerns the studies related to a constant viscosity case, or a few works focus on either temperature-dependent viscosity or magnetic-field dependent viscosity again for the linear case of ferroconvection. But we note that one of the distinct thermo-physical property viz., viscosity of a fluid which varies with temperature and magnetic field finds its importance in technological applications among others. Also, to accurately predict the flow and heat transport rates it is necessary to take into account the variation of viscosity. To the best of our knowledge, the literature on modulation with

variable viscosity is virtually non-existent.

As seen in the literature the non-sinusoidal waveforms modeled by an infinite Fourier series makes a significant contribution in understanding the heat transport in a fluid. This serves as a motivation for us to study the waveforms. The unanswered questions of efficiency of using non-sinusoidal waveform when exposed to a modulation in ferrofluids makes it necessary to study the effect of ferroconvection on the heat transport performance. Thus, considering the practical applications using ferrofuids, thermorheological and magnetorheological effects are given attention by treating the effective viscosity as a function of both temperature and magnetic field strength in contrast to the various problems of convection with constant viscosity. As ferrofluids with variable viscosity can have a significant change within a few millisecond when an external magnetic field is applied making it a problem worth considering for applications owing to engineering and medicine.

Chapter 4

Objective

The aim of the present work is to study the influence of thermorheological and magnetorheological effects on ferroconvection under gravity, rotation, thermal and magnetic modulations for the sinusoidal and non-sinusoidal waveforms. The objectives in an ellaborate sense can be enumerated as follow:

1. To comprehend the pre onset and post onset regimes of RBF with variable viscosity under different modulations.

2. To study the effects of thermorheology and magnetorheology on the RBF.

3. To comprehend the effects of the time dependent body force, coriolis force, time depedndent magnetic field, and the synchronous, lower wall and the asynchronous boundary temperature modulations on RBF with variable viscosity.

4. To control heat transport rates more efficiently using sinusoidal and non sinusoidal waveforms.

Thus, the study paves way in understanding the effects of magnetism, variable viscosity, modulations and waveforms on onset of convection and amount of heat transport in ferrofluids. As the unison of these factors would significantly contribute in achieving desired amounts of heat in a system for an application oriented study. Further, the work being analytical helps in arriving at the most accurate predictions on the onset and post-onset regimes of Rayleigh-Bénard ferroconvection.

Chapter 5

Methodology

We theoretically explore the dynamics of a ferrofluid with temperature and magnetic field dependent viscosity which is in a Rayleigh-Bénard situation and is subjected to gravity, rotation, thermal and magnetic modulations. The problem considers both sinusoidal and non-sinusoidal time-periodic variations to study the onset and post-onset regimes of Rayleigh-Bénard ferroconvection. We adopt the systematic procedures of Venezian and Lorenz to study the linear and non-linear stability analyses respectively. The perturbations in the system due to external reasons are expanded in terms of the amplitude of modulation in the case of linear stability and truncated representation of the Fourier series in the case of non-linear stability. However we note a difference in the procedures to arrive at the expressions for the critical Rayleigh number, R_c and the correction Rayleigh number, R_{2c}. For the gravity, rotation and magnetic modulations a non-autonomous Lorenz model for the problem is first obtained and both linear and non-linear analyses of the system is performed using it. Then R_c and R_{2c} is found from the linearized Lorenz model. The Lorenz system of equations is solved for the amplitude to arrive at the Nusselt number, (Nu) which quantifies the heat transport. However the thermal modulation problem follows the usual procedure of Venezian where we first arrive at the expressions for R_c and R_{2c} and then by using the truncated representation of the Fourier series we obtain the Lorenz model.

Chapter 6

Overview of the Problems

This study focuses on the following four problems (discussed in Chapter 7 in detail):

Problem 1:

Thermoconvective Instability in a Vertically Oscillating Horizontal Ferrofluid Layer with Variable Viscosity

Problem 2:

Effects of Variable Viscosity and Rotation Modulation on Ferroconvection

Problem 3:

Effects of Synchronous, Lower Wall and Asynchronous Boundary Temperature Modulations in a Ferrofluid with Variable Viscosity for Sinusoidal and Non-Sinusoidal Waveforms

Problem 4:

Effect of Time-Periodic Magnetic Modulation on Rayleigh-Bénard Ferro-Convection with Variable Viscosity

The dimensionless parameters appearing in this study are the following:

(i) Thermal Rayleigh number, R

$$R = \frac{\alpha g \Delta T d^3 \rho_0}{\mu_0 \kappa},$$

(ii) Prandtl number, Pr

$$Pr = \frac{\mu}{\rho_0 \kappa},$$

(iii) Buoyancy magnetic umber, M_1

$$M_1 = \frac{\mu_0 K_l^2 \Delta T}{\alpha g \rho_0 (1+\chi_m) d},$$

(iv) Non-buoyancy magnetic number, M_3

$$M_3 = \frac{1+M_0/H_0}{1+\chi_m},$$

(v) Variable viscosity parameter, V

$$V = \left(\delta_T - \frac{\delta_H K_l}{1+\chi_m}\right)\Delta T,$$

(vi) Taylor number, Ta

$$Ta = \frac{4\rho^2 d^4}{\kappa^2},$$

(vii) Chandrasekhar number, Q

$$Q = \frac{\mu_m H_0^2 d^2}{\nu_m \mu}$$

(viii) Nusselt number, Nu

$$Nu(t) = \frac{Heat\ transport\ by\ (conduction + convection)}{Heat\ transport\ by\ (conduction)} = \frac{H_T}{(\kappa \Delta T/d)}$$

Nomenclature

Greek Symbols

α	Thermal expansion coefficient
χ	Thermal diffusivity
χ_m	Magnetic susceptibility
ΔT	Temperature difference
δ	Amplitude of modulation
δ_T, δ_H	Small positive constants
γ_m	Magnetic viscosity
κ	Thermal conductivity
μ	Coefficient of Viscosity
$\mu(H,T)$	Variable viscosity
μ_m	Magnetic permeability
∇^2	Two dimensional Laplacain operator $\left(=\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial z^2}\right)$
ω	Frequency of modulation
Φ	Phase angle
ϕ	Magnetic potential
ψ	Stream function
ρ	Density of the ferrofluid

$\vec{\Omega}$ Speed of rotation

ζ Vorticity

Latin Symbols

$\hat{k}$ Unit vector in the z direction

$\mathcal{A}$ Area under the curve

$\vec{B}$ Magnetic induction

$\vec{g} = (0,0,-g)$ Gravitational acceleration (ms^{-2})

$\vec{H}$ Magnetic field

$\vec{M}$ Magneization

$\vec{q} = (u,0,w)$ velocity vector

$A_1(t), A_2(t) ... A_7(t)$ Amplitudes of convection

C_{VH} specific heat at constant volume and magnetic field ($Jkg^{-1}K^{-1}$)

d Depth of the horizontal plates (m)

H_0 Applied uniform vertical magnetic field

h_1, h_2 Jacobian terms

J Jacobian of a matrix

k Wave number

K_l Pyromagnetic coefficient

L Operator

M_0 Mean value of magnetization at $H = H_0$ and $T = T_0$

M_1 Buoyancy magnetization number

M_3 Non-buoyancy magnetization number

Nu Nusselt number

p Pressure

Pm	Magnetic Prandtl number
Pr	Prandtl number
Q	Chandrasekhar number
R	Rayleigh number
$r(\omega,t)$	time dependent modulated rotation
S	Region of interest
T	Temperature
t	Time
Ta	Taylor number
u,w	Components of velocity along x and z directions respectively
V	Variable viscosity parameter
W_j	Operator (where $j = 0,1,2$)

Subscripts

0	At reference value
b	basic state
c	Critical
SqW	Square wave
STW	Sawtooth wave
SW	Sinusoidal wave
TW	Triangular wave

Superscripts

$'$	perturbed quantity
$*$	dimensionless quantity
Tr	Transpose

Chapter 7

Research Problems

7.1 Thermoconvective Instability in a Vertically Oscillating Horizontal Ferrofluid Layer with Variable Viscosity

Mathematical Formulation

An infinite extent, horizontal layer of Newtonian Boussinesq ferrrofluid with temperature and magnetic-field-dependent viscosity, $\mu(H,T)$, of finite depth d in the region of interest $C = \{(x,z)/-\infty < x < \infty, 0 \leq z \leq d\}$ is considered. The upper and lower boundaries are maintained at constant temperatures T_0 and $T_0 + \Delta T$ ($\Delta T > 0$) respectively. The geometrical configuration considered is shown in Fig. 1.

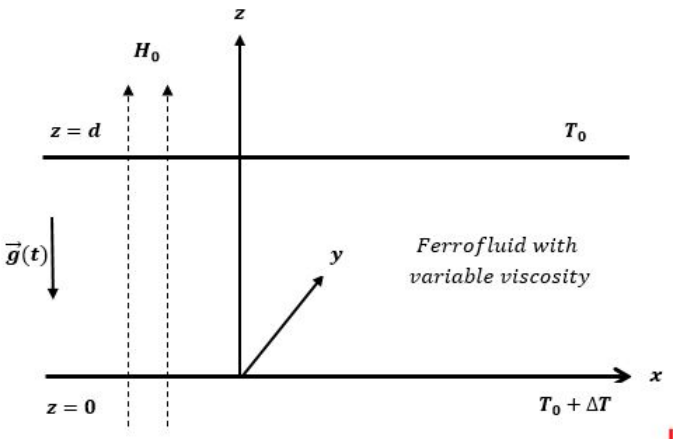


Figure 7.1: Physical configuration of Ferroconvection under gravity modulation

An uniform transverse magnetic field H_0 is applied in the vertical z-direction and the body force is varying with time, $\vec{g} = (0,0,g(t))$, where $g(t) = -g_0(1+\delta F_i(\omega,t))$, δ is small amplitude and $F_i(\omega,t)$ is the waveform of modulation with $i = 1,2,3,4$. Furthermore stress-free and isothermal boundaries are considered. The governing system of equations [80, 48] to study ferroconvection with thermorheological and magnetorheological effects under the Boussinesq approximation are:

Conservation Equation of Mass

$$\nabla.\vec{q} = 0, \tag{7.1.1}$$

Conservation Equation of Linear Momentum

$$\rho_0\left[\frac{\partial\vec{q}}{\partial t} + (\vec{q}.\nabla)\vec{q}\right] = -\nabla p + \rho(T)\vec{g}\,[1+\delta F_i(\omega,t)] + \mu_0(\vec{M}.\nabla)\vec{H} + \nabla.\Big[\mu(H,T)(\nabla\vec{q} + \nabla\vec{q}^{Tr})\Big], \tag{7.1.2}$$

Conservation Equation of Energy

$$\rho_0 C_{VH}\left[\frac{\partial T}{\partial t}+(\vec{q}.\nabla)T\right]=\kappa\nabla^2 T, \tag{7.1.3}$$

Density Equation Of State

$$\rho=\rho_0[1-\alpha(T-T_0)], \tag{7.1.4}$$

Maxwell Equations

$$\nabla.\vec{B}=0 \qquad \nabla\times\vec{H}=0, \tag{7.1.5}$$

Magnetic Equation of State

$$M=M_0+\chi_m(\vec{H}-H_0)-K_1(T-T_0), \tag{7.1.6}$$

Effective Viscosity [53]

$$\mu(H,T)=\frac{\mu_0}{1+\delta_T(T-T_0)-\delta_H(\vec{H}-H_0)}. \tag{7.1.7}$$

The non-dimensional equations which govern the disturbances when superposed over the basic state take the following form after the introduction of the stream function, ψ in the form $u=-\frac{\partial\psi}{\partial z}, w=\frac{\partial\psi}{\partial x}$, introduction of ϕ, in the form $H_x=\frac{\partial\phi}{\partial z}, H_z=-\frac{\partial\phi}{\partial x}$, and after dropping the asterisks:

$$\begin{aligned}\frac{1}{Pr}\frac{\partial}{\partial t}(\nabla^2\psi)=G_1(z)(\nabla^4\psi)+2DG_1(z)(\nabla^2\psi)-D^2G_1(z)\left(\frac{\partial^2\psi}{\partial x^2}-\frac{\partial^2\psi}{\partial z^2}\right)\\ -R[1+\delta F_i(\omega,t)]\frac{\partial T}{\partial x}-RM_1\left(\frac{\partial^2\phi}{\partial x\partial z}-J\left(T,\frac{\partial\phi}{\partial z}\right)-\frac{\partial T}{\partial x}\right)+\frac{1}{Pr}J(\psi,\nabla^2\psi),\end{aligned} \tag{7.1.8}$$

$$\left(\frac{\partial}{\partial t}-\nabla^2\right)T=-\frac{\partial\psi}{\partial x}+J(\psi,T), \tag{7.1.9}$$

$$M_3\frac{\partial^2\phi}{\partial x^2}+\frac{\partial^2\phi}{\partial z^2}=\frac{\partial T}{\partial z}, \tag{7.1.10}$$

where

$$D=\frac{\partial}{\partial z}, \quad \nabla^2=\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial z^2}, \quad G_1(z)=\frac{1}{1+f(z)V}, \quad f(z)=\left(1-\frac{z}{d}\right). \tag{7.1.11}$$

In this study we use four types of time-periodic oscillations [98] ,viz,

Case (i) g-jitter with Square Waveform (with i=1):

$$g(\omega,t)=4\delta F_1(\omega,t)=\frac{4\delta}{\pi}\sum_{n=1,3,5,..}^{\infty}\frac{1}{n}sin(n\omega t).$$

Case (ii) g-jitter with Sinusoidal Waveform (with i=2):

$g(\omega,t) = \delta F_2(\omega,t) = \delta sin(\omega t).$

Case (iii) g-jitter with Triangular Waveform (with i=3):

$$g(\omega,t) = 8\delta F_3(\omega,t) = \frac{8\delta}{\pi^2} \sum_{n=1,3,5,..}^{\infty} \frac{(-1)^{\frac{n-1}{2}}}{n^2} sin(n\omega t).$$

Case (iv) g-jitter with Sawtooth Waveform (with i=4):

$$g(\omega,t) = 4\delta F_4(\omega,t) = \frac{4\delta}{n^2\pi^2}[\big(1 - cos(n\pi)\big)cos(n\omega t)].$$

The Eqns. (7.1.8) - (7.1.10) are solved subject to the stress-free, isothermal, and magnetic boundary conditions based on the assumption of infinite magnetic susceptibility, in the form

$$\psi = \frac{\partial^2 \psi}{\partial z^2} = T = \frac{\partial \phi}{\partial z} = 0. \quad at \quad z = 0, 1. \tag{7.1.12}$$

The exact solution of the convective instability problem in a ferromagnetic fluid for free-free, isothermal boundaries was discussed by Finlayson [25].

7.1.1 Non-Linear Stability Analysis

In what follows in this section, we derive the generalized magnetic-Lorenz model by using a minimal mode truncated Fourier series representation with the study restricted to two-dimensional flows with the velocity field expressed in terms of stream function, ψ. Using the idealized boundary conditions (7.1.12) a weakly non-linear stability analysis of the system (7.1.8)-(7.1.10) is performed by expanding the functions ψ, T and ϕ as below:

$$\psi = A_1(t)sin(\pi\alpha x)sin(\pi z), \tag{7.1.13}$$

$$T = A_2(t)cos(\pi\alpha x)sin(\pi z) + A_3(t)sin(2\pi z), \tag{7.1.14}$$

$$\phi = A_4(t)cos(\pi\alpha x)cos(\pi z) + A_5(t)cos(2\pi z), \tag{7.1.15}$$

We obtain the following set of ordinary differential equations for the time evolution of the amplitudes by substituting Eqns. (7.1.13)-(7.1.15) in Eqns. (7.1.8)-(7.1.10):

$$\begin{aligned} \dot{A_1}(t) = -\frac{Pr}{k^2} f(V) A_1(t) + \left[\frac{\pi\alpha PrRM_1}{k^2(1+M_3\alpha^2)} - \frac{\pi\alpha PrR}{k^2}[1 + \delta F_i(\omega,t) + M_1]\right] A_2(t) \\ + \frac{\pi^2\alpha PrRM_1}{k^2}\left[\frac{1}{(1+M_3\alpha^2)} - 1\right] A_2(t)A_3(t), \end{aligned} \tag{7.1.16}$$

$$\dot{A_2}(t) = -\pi\alpha A_1(t) - \pi^2\alpha A_1(t)A_3(t) - k^2 A_2(t), \tag{7.1.17}$$

$$\dot{A_3}(t) = -4\pi^2 A_3(t) + \frac{\pi^2\alpha}{2} A_1(t)A_2(t), \tag{7.1.18}$$

where,

$$\left.\begin{aligned} k^2 = \pi^2(1+\alpha^2), \qquad f(V) = \left[v_1 k^4 + 2v_2 k^2 - v_3\pi^2(1-\alpha^2)\right] \\ v_1 = \int_0^1 G_1(z)sin^2(\pi z)dz, \qquad v_2 = \int_0^1 DG_1(z)sin^2(\pi z)dz, \qquad v_3 = \int_0^1 D^2G_1(z)sin^2(\pi z)dz. \end{aligned}\right\} \tag{7.1.19}$$

7.1.2 Linear Stability Analysis

Prior to performing the linear stability analysis we note that in the problem the principle of exchange of stabilities holds good In this section, we perform a linear stability analysis by using the linearized form of the magnetic-Lorenz Eqns. (7.1.16)-(7.1.18) and obtain the expression of the eigen value for the problem using the Venezian method [115] and following the procedure of [98].

The linearized version of Lorenz Eqns. (7.1.16)-(7.1.18) are:

$$\frac{dA_1}{dt} = -\frac{Pr}{k^2} f(V)A_1(t) + \left[\frac{\pi\alpha PrRM_1}{k^2(1+M_3\alpha^2)} - \frac{\pi\alpha PrR}{k^2}[1+\delta F_i(\omega,t)+M_1]\right] A_2(t), \tag{7.1.20}$$

$$\frac{dA_2}{dt} = -\pi\alpha A_1(t) - k^2 A_2(t), \tag{7.1.21}$$

$$\frac{dA_3}{dt} = -4\pi^2 A_3(t). \tag{7.1.22}$$

Gravity modulation is contemplated to be of $O(\delta)$ and we thus expand the amplitudes $A_1(t)$, $A_2(t)$, $A_3(t)$ and the Rayleigh number, R, in the form:

$$\left.\begin{aligned} A_1 &= A_{10} + \delta A_{11} + \delta^2 A_{12} + \delta^3 A_{13} + ... \\ A_2 &= A_{20} + \delta A_{21} + \delta^2 A_{22} + \delta^3 A_{23} + ... \\ A_3 &= A_{30} + \delta A_{31} + \delta^2 A_{32} + \delta^3 A_{33} + ... \\ R &= R_c + \delta R_1 + \delta^2 R_2 + \delta^3 R_3 + ... \end{aligned}\right\} \tag{7.1.23}$$

Thus following the procedure of Siddheshwar and Kanchana [98] we obtain the following:

Unmodulated critical Rayleigh number, R_c

$$R_c = \frac{k^6}{\pi^2\alpha^2}\frac{f(V)}{M}, \tag{7.1.24}$$

where

$$M = \frac{1+M_3\alpha^2+M_1M_3\alpha^2}{1+M_3\alpha^2}. \tag{7.1.25}$$

Correction Rayleigh number, R_{2c}

$$R_{2c} = Re\left\{\frac{{R_c}^2 Pr\pi\alpha(Y_1 - iY_2)}{2k^2 M(Y_1^2 + Y_2^2)}\right\}, \tag{7.1.26}$$

where Re denotes the real part and

$$Y_1 = \frac{\omega^2}{\pi\alpha} - \frac{Prf(V)}{\pi\alpha} + \frac{R_c Pr\pi\alpha}{k^2}M, \quad Y_2 = \frac{\omega k^2}{\pi\alpha} + \frac{Prf(V)}{k^2\pi\alpha}. \tag{7.1.27}$$

Estimation of Heat Transport

Predominantly the focus of the nonlinear part of the study is on the heat transport which is represented by the Nusselt number, Nu and is given by

$$Nu(t) = 1 + \frac{\left[\frac{k_c}{2\pi}\int_0^{\frac{2\pi}{k_c}} \frac{\partial T}{\partial z}dx\right]_{z=0}}{\left[\frac{k_c}{2\pi}\int_0^{\frac{2\pi}{k_c}} \frac{dT_b}{dz}dx\right]_{z=0}}. \tag{7.1.28}$$

On substituting for T x and T_b from the basic state and simplifying we obtain the following expression for the Nusselt number

$$Nu(t) = 1 - 2\pi A_3(t). \tag{7.1.29}$$

where $A_3(t)$ is calculated by solving the magnetic-Lorenz model in Eqns. (7.1.16)-(7.1.18).
Further, we determine the average Nusselt number using the following integral,

$$\overline{Nu(t)} = \frac{\omega}{2\pi}\int_0^{\frac{2\pi}{\omega}} Nu(t)dt. \tag{7.1.30}$$

Table 7.1: Values of the critical Rayleigh Number (R_c) and critical wave number (α_c) for the limiting case of constant viscosity, nonmagnetic fluids in the absence of gravity modulation.

	Platten and Legros [60]	**Siddheshwar et.al [103]**	**Sekhar et.al [81]**	**Present Problem**
R_c	657.511	657.592	657.514	657.511
α_c	2.22	2.225	2.221	2.221

7.1.3 Results

1. The parameters V, M_1 and M_3 have opposite influences on the critical Rayleigh number, R_c, compared to that on the Nusselt number, Nu. This is in accordance with the physical interpretation that there will be a maximum heat transport in the system when the ferrofluids sets in to convection mode faster.

Table 7.2: Values of (R_c) and critical wave Number (α_c) for ferroconvection in the absence of gravity modulation, for $M_1 = M_3 = 1$

	$V = 0$			$V = 0.5$	
	Finlayson [25]	**Sekhar et.al [81]**	**Present Problem**	**Sekhar et.al [81]**	**Present Problem**
R_c	482.903	482.903	482.903	375.538	377.738
α_c	2.51404	2.51404	2.51404	2.49847	2.4857

Table 7.3: Values of $\overline{Nu}$ for the classical case $(V = 0, M_1 = M_3 = 0)$ in the presence of gravity modulation.

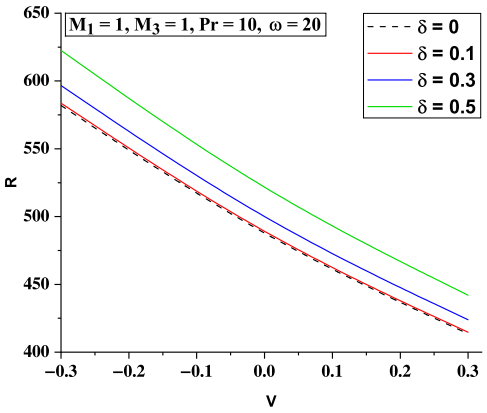
	With Modulation, $\delta = 0.2$			
	SqW	**SW**	**TW**	**STW**
Siddheshwar and Kanchana[98]	2.064	2.042	2.033	-
Present Problem	2.064	2.043	2.034	2.031

2. The threshold of onset as marginal stationary convection in a ferrofluid with thermorheological and magnetorheological effects can be regulated by appropriately tuning the frequency of gravity modulation. This aspect can be exploited to control convection depending on the need of an application involving smart liquids like the ferrofluids.

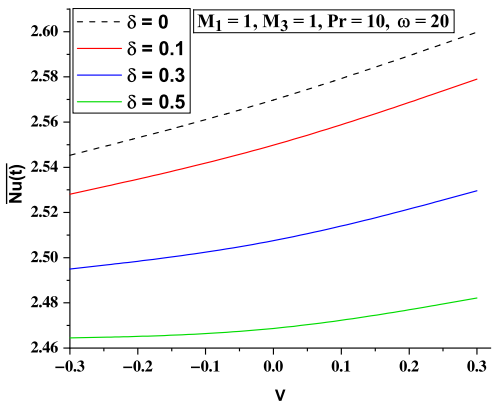
3. Compared to the modulated problem the no modulation one is the most stable.

4. The thermorheological and magnetorheological effects are antagonistic in nature and hence serve as two external mechanisms for regulating convection and thereby heat transport.

5. The square waveform facilitates maximum heat transport in the system while the sawtooth waveform facilitates least heat transport. The former influences a greater part of the fluid system compared to the latter and hence this observed result.

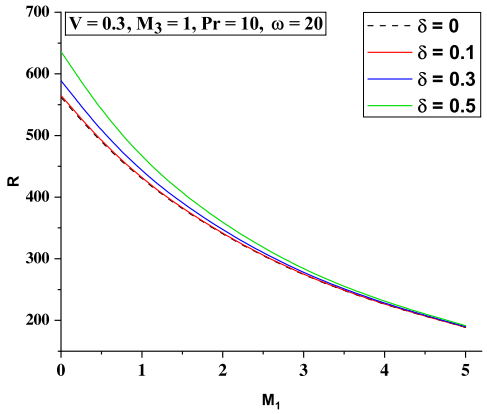


a: Plot of Rayleigh number, $R = R_c + \delta^2 R_{2c}$ vs. V.

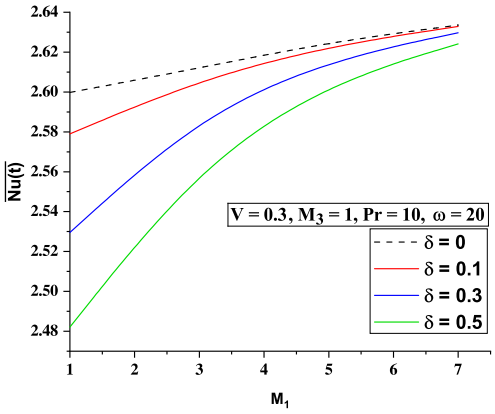


b: Plot of average Nusselt number, $\overline{Nu(t)}$ vs. V

Figure 7.2: Plots of R and $\overline{Nu(t)}$ for varying V.

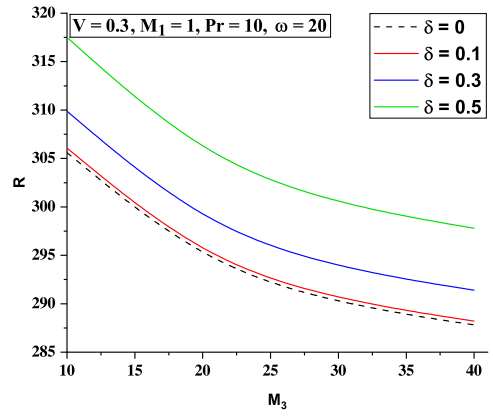


a: Plot of R vs. M_1.

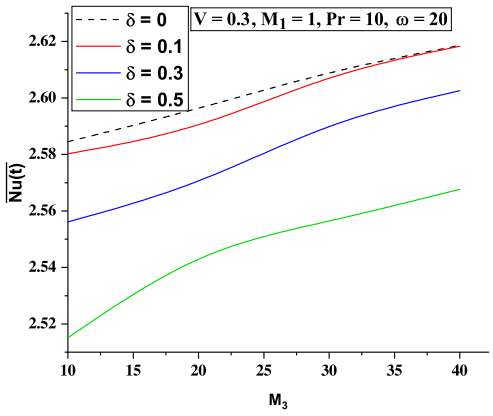


b: Plot of average $\overline{Nu(t)}$ vs. M_1

Figure 7.3: Plots of R and $\overline{Nu(t)}$ for varying M_1.

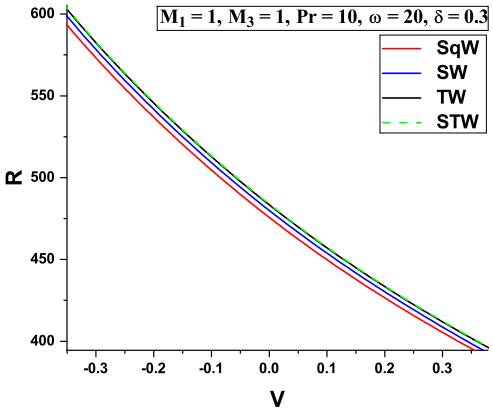


a: Plot of R vs. M_3.

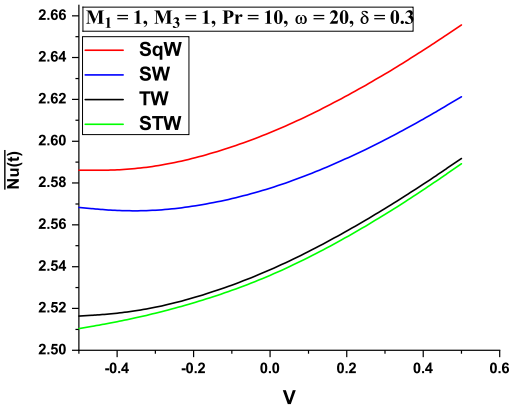


b: Plot of average $\overline{Nu(t)}$ vs. M_3

Figure 7.4: Plots of R and $\overline{Nu(t)}$ for varying M_3.

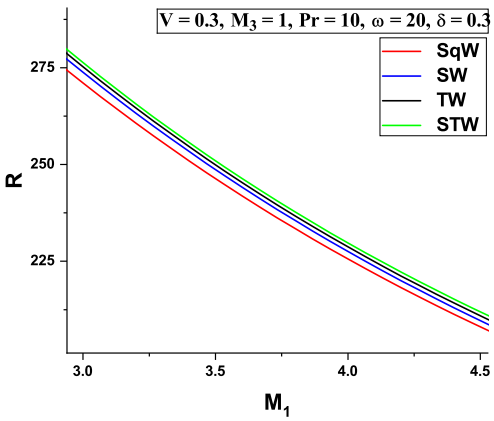


a: Plot of R vs. V.

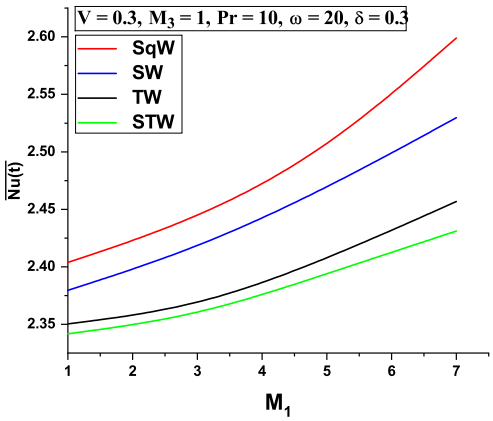


b: Plot of average $\overline{Nu(t)}$ vs. V.

Figure 7.5: Plots of R and $\overline{Nu(t)}$ vs. V for different waveforms.

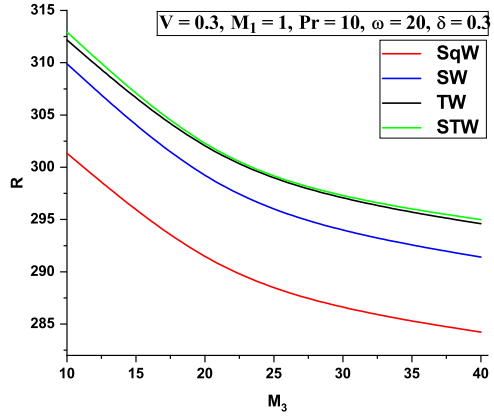


a: Plot of R vs. M_1.

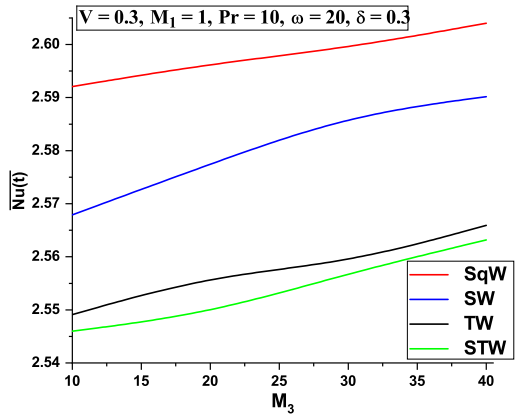


b: Plot of average $\overline{Nu(t)}$ vs. M_1

Figure 7.6: Plots of R and $\overline{Nu(t)}$ vs. M_1 for different waveforms.



a: Plot of R vs. M_3.



b: Plot of average $\overline{Nu(t)}$ vs. M_3

Figure 7.7: Plots of R and $\overline{Nu(t)}$ vs. M_3 for different waveforms.

7.2 Effects of Variable Viscosity and Rotation Modulation on Ferroconvection

Mathematical formulation

The geometrical configuration considered for this problem has a similar setup of the problem 7.1 along with an uniform transverse magnetic field H_0 being applied in the vertical z-direction and the speed of rotation is assumed to be in sinusoidal and non-sinsoidal variations in relation to time and given as $\vec{\Omega}(t) = \Omega_0(1+\delta F_i(\omega,t))\hat{k}$ where, $F_i(\omega,t)$ is the waveform of modulation with $i = 1,2,3,4$ for the SqW, SW, TW and STW respectively. Furthermore stress-free and isothermal boundaries are considered. The schematic of this problem is shown in Fig. 7.8.

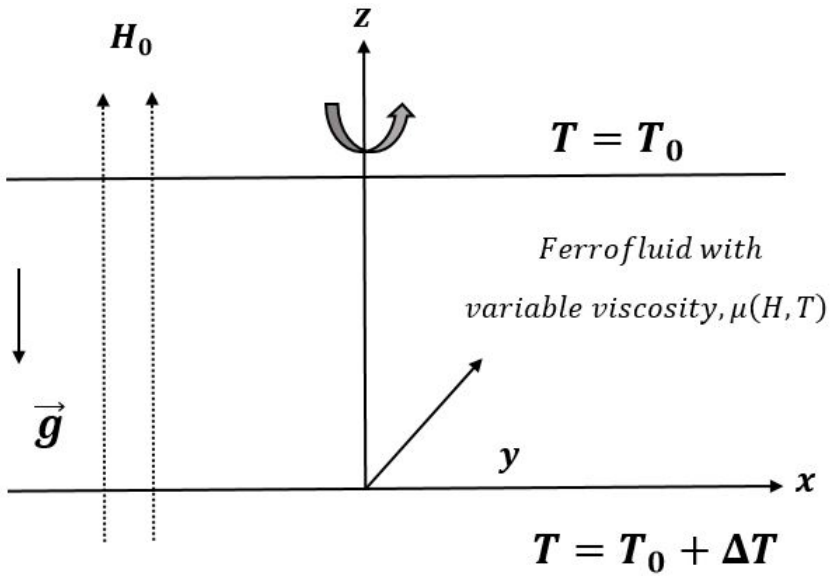


Figure 7.8: Physical configuration of Ferroconvection under rotation modulation

The governing system of equations [80], [48] to study ferroconvection with thermorheological and magnetorheological effects along with the effect of rotation under the Boussinesq approximation are:

Conservation Equation of Mass

$$\nabla.\vec{q} = 0, \tag{7.2.1}$$

Conservation Equation of Linear Momentum

$$\rho_0\left[\frac{\partial\vec{q}}{\partial t} + (\vec{q}.\nabla)\vec{q} + 2\overrightarrow{\Omega(t)}\times\vec{q}\right] = -\nabla p + \rho\vec{g} + \mu_0(\vec{M}.\nabla)\vec{H} + \nabla.\Big[\mu(H,T)(\nabla\vec{q} + \nabla\vec{q}^{Tr})\Big], \tag{7.2.2}$$

Conservation Equation of Energy

$$\rho_0 C_{VH}\left[\frac{\partial T}{\partial t}+(\vec{q}.\nabla)T\right]=\kappa\nabla^2 T, \tag{7.2.3}$$

Density Equation Of State

$$\rho=\rho_0[1-\alpha(T-T_0)], \tag{7.2.4}$$

Maxwell Equations

$$\nabla.\vec{B}=0 \qquad \nabla\times\vec{H}=0, \tag{7.2.5}$$

Magnetic Equation of State

$$M=M_0+\chi_m(H-H_0)-K_1(T-T_0), \tag{7.2.6}$$

Effective Viscosity [53]

$$\mu(H,T)=\frac{\mu_0}{1+\delta_T(T-T_0)-\delta_H(H-H_0)}. \tag{7.2.7}$$

The non-dimensional equations which govern the disturbances when superposed over the basic state take the following form after after eliminating the pressure, introducing ψ in the form $u=-\frac{\partial\psi}{\partial z}, w=\frac{\partial\psi}{\partial x}$, introduction of ϕ, in the form $H_x=\frac{\partial\phi}{\partial z}$, $H_z=-\frac{\partial\phi}{\partial x}$, and after dropping the asterisks:

$$\begin{aligned}\frac{1}{Pr}\frac{\partial}{\partial t}(\nabla^2\psi)&=G_1(z)(\nabla^4\psi)+2DG_1(z)(\nabla^2\psi)-D^2G_1(z)\left(\frac{\partial^2\psi}{\partial x^2}-\frac{\partial^2\psi}{\partial z^2}\right)-R\frac{\partial T}{\partial x}\\&+\sqrt{Ta}[1+\delta F_i(\omega,t)]\frac{\partial\zeta}{\partial z}-RM_1\left(\frac{\partial^2\phi}{\partial x\partial z}-J\left(T,\frac{\partial\phi}{\partial z}\right)-\frac{\partial T}{\partial x}\right)+\frac{1}{Pr}J(\psi,\nabla^2\psi),\end{aligned} \tag{7.2.8}$$

$$\left[\frac{1}{Pr}\frac{\partial}{\partial t}\nabla^2\right]\zeta=-\sqrt{Ta}[1+\delta F_i(\omega,t)]\frac{\partial\psi}{\partial z}+\frac{1}{Pr}J(\psi,\zeta) \tag{7.2.9}$$

$$\left(\frac{\partial}{\partial t}-\nabla^2\right)T=-\frac{\partial\psi}{\partial x}+J(\psi,T), \tag{7.2.10}$$

$$M_3\frac{\partial^2\phi}{\partial x^2}+\frac{\partial^2\phi}{\partial z^2}=\frac{\partial T}{\partial z}, \tag{7.2.11}$$

where

$$f(z)=\left(1-\frac{z}{d}\right), \qquad G_1(z)=\frac{1}{1+f(z)V}. \tag{7.2.12}$$

The Eqns. (7.2.8) - (7.2.11) are solved subject to the stress-free, isothermal, and magnetic boundary conditions based on the assumption of infinite magnetic susceptibility, in the form

$$\psi = \frac{\partial^2 \psi}{\partial z^2} = \frac{\partial \zeta}{\partial z} = T = \frac{\partial \phi}{\partial z} = 0 \quad at \quad z = 0, 1. \tag{7.2.13}$$

7.2.1 Non-Linear Stability Analysis

Using the idealized boundary conditions (7.2.13) a weakly non-linear stability analysis of the system (7.2.8)-(7.2.11) is performed by expanding the functions ψ, ζ, T and ϕ as follows

$$\psi = A_1(t) sin(\pi\alpha x) sin(\pi z), \tag{7.2.14}$$

$$\zeta = A_2(t) sin(\pi\alpha x) cos(\pi z) + A_3(t) sin(2\pi\alpha x) sin(\pi z), \tag{7.2.15}$$

$$T = A_4(t) cos(\pi\alpha x) sin(\pi z) + A_5(t) sin(2\pi z), \tag{7.2.16}$$

$$\phi = A_6(t) cos(\pi\alpha x) cos(\pi z) + A_7(t) cos(2\pi z), \tag{7.2.17}$$

On simplification we obtain the following set of ordinary differential equations for the time evolution of the amplitudes, A_i's for the ferrofluid convection with temperature and magnetic-field-dependent viscosity under rotation modulation:

$$\begin{aligned} \dot{A_1}(t) = &-\frac{Pr}{k^2} f(V) A_1(t) - \frac{\pi\alpha PrR}{k^2}(1+M_1) A_4(t) + \frac{\pi\alpha PrRM_1}{k^2(1+M_3\alpha^2)} A_4(t) \\ &\frac{\pi Pr}{k^2}\sqrt{Ta}(1+\delta F_i(\omega,t)) A_2(t) + \frac{\pi^2\alpha PrRM_1}{k^2}\left[\frac{1}{(1+M_3\alpha^2)} - 1\right] A_4(t) A_5(t), \end{aligned} \tag{7.2.18}$$

$$\dot{A_2}(t) = -k^2 Pr A_2(t) - \pi Pr\sqrt{Ta}(1+\delta F_i(\omega,t)) A_1(t) + \pi^2\alpha A_1(t) A_3(t), \tag{7.2.19}$$

$$\dot{A_3}(t) = -\frac{\pi^2\alpha}{2} A_1(t) A_2(t) - 4\pi^2\alpha^2 Pr A_3(t), \tag{7.2.20}$$

$$\dot{A_4}(t) = -\pi\alpha A_1(t) - \pi^2\alpha A_1(t) A_5(t) - k^2 A_4(t), \tag{7.2.21}$$

$$\dot{A_5}(t) = -4\pi^2 A_5(t) + \frac{\pi^2\alpha}{2} A_1(t) A_4(t), \tag{7.2.22}$$

We study the efficiency in the heat transport results by using the following integral which determines the average Nusselt number,

$$\overline{Nu(t)} = \frac{\omega}{2\pi}\int_0^{\frac{2\pi}{\omega}} Nu(t) dt. \tag{7.2.23}$$

where,

$$Nu(t) = 1 - 2\pi A_5(t). \tag{7.2.24}$$

7.2.2 Linear Stability Analysis

Following a similiar procedure of the gravity modulation problem 7.1 we arrive at the following:

Unmodulated critical Rayleigh number

$$R_c = \frac{k^2 f(V) + \pi^2 Ta}{\pi^2 \alpha^2 M}. \tag{7.2.25}$$

Correction Rayleigh number, R_{2c}

The solution of the second order gives the correction Rayleigh number, R_{2c} which takes the form,

$$R_{2c} = \frac{\pi k^2 Pr^2 Ta^2 \delta_1^3}{2\alpha^3 (Y_1^2 + Y_2^2)} \left[\frac{(N_1 Y_1 - N_2 Y_2)}{\omega^2 + k^4 Pr^2} \right] N_3 + \frac{k^4 Pr^2 Ta \delta_1^3}{2\alpha^2 (\omega^2 + k^4 Pr^2)} + \frac{\pi k^2 Pr Ta^2 \delta_1^2}{2\alpha^2 (Y_1^2 + Y_2^2)} \left[-k^2 Y_1 + \omega Y_2 \right] N_3, \tag{7.2.26}$$

$$Y_1 = \frac{\omega^2}{\pi\alpha} - \frac{Pr}{\pi\alpha} f(V) - \frac{\pi Pr^2 Ta}{k^2 \alpha} \left(\frac{N_1}{\omega^2 + k^4 Pr^2} \right) + \frac{\pi \alpha Pr R_c M}{\pi \alpha k^2}, \tag{7.2.27}$$

$$Y_2 = \frac{\omega k^2}{\pi\alpha} + \frac{Pr f(V) \omega}{\pi \alpha k^2} - \frac{\pi Pr^2 Ta}{k^2 \alpha} \left(\frac{-\omega k^2 Pr + \omega k^2}{\omega^2 + k^4 Pr^2} \right), \tag{7.2.28}$$

$$N_1 = \omega^2 + k^4 Pr \tag{7.2.29}$$

$$N_2 = \omega k^2 + Pr \tag{7.2.30}$$

$$N_3 = \frac{Pr^2}{\omega^2 + k^4 Pr^2} + \frac{1}{k^4} \tag{7.2.31}$$

$Ta = 10, V = 0, M_1 = 0, M_3 = 0$			
Authors	Chandrasekhar [20]	Bhadauria [14]	Present work
Description of the convection problem	Instability in a fluid layer under the influence of rotation and magnetic field	Effect of rotation modulation in a non-magnetic, temperature dependent viscosity fluid	Effect of rotation modulation in a ferrofluid with variable viscosity
R_c	677.077	677.077	677.077
a_c	2.27012	2.27012	2.27012
Effect of rotation	Delays the onset of convection		

Table 7.4: Comparison of results of present work with that of previous authors' works for a RBC problem with the effect of rotation

$Ta = 10, V = 0, M_1 = 1, M_3 = 1$				
Authors	**Description of the convection problem**	R_c	a_c	**Effect of rotation on ferroconvection**
Siddheshwar et al. [101]	Theoretical investigation of the onset of Ferroconvection subjected to rotation	493.892	2.56064	Delays the onset of convection
Present problem	Effect of rotation modulation in a ferrofluid with variable viscosity	493.892	2.56064	

Table 7.5: Comparison of results of present work with that of Siddheshwar et al. [101] for the effect of rotation on ferroconvection with constant viscosity

7.2.3 Results

This study investigates the effects of rotation modulation on the convective stability of ferrofluid with variable viscosity. The results are presented below,

1. $R_c|_{V\neq 0} < R_c|_{V=0}, \quad \overline{Nu(t)}|_{V\neq 0} > \overline{Nu(t)}|_{V=0}$
2. The stabilizing effect of V is found to reverse as the modulating frequency increases. The results show that at $\omega \approx 140$ the system reverses from stability to instability.
3. The impact of Taylor number is to stabilize the system.
4. The effect of average Nusselt number, $\overline{Nu(t)}$ can be summarized as follows
   (a) $\overline{Nu(t)}|_{V<0} < \overline{Nu(t)}|_{V=0} < \overline{Nu(t)}|_{V>0}$.
   (b) $\overline{Nu(t)}$ increases with increasing M_1 and M_3.
   (c) $\overline{Nu(t)}_{TW} < \overline{Nu(t)}_{STW} < \overline{Nu(t)}_{SW} < \overline{Nu(t)}_{SqW}$.
5. The thermorheological and magnetorheological effects are antagonistic in nature with the thermorheological effect shooting up the transport of heat leading to an advanced setting in of convection and thus making the system unstable. On the contrary the magnetorheological effect being a counter-force slows down the fluid motion, thus decreasing the transport of heat and thereby postponing the setting in of convection by giving rise to a stable system.

6. The increase in M_3 increases the departure of linearity in the magnetic equation of state resulting in an enhanced heat transport. This furnishes an unstable system owing to the increase in the velocity of the fluid perpendicular to the planes along with the release of excess energy arising due to magnetization and the thermal energy. Thus,M_3 destabilizes the system. Furthermore M_1 also destabilizes the system.

7. Streamlines attain their maximum size beyond $t = 0.5$, i,e the sytem achieves its steady state.

Thus we conclude that in a ferrofluid with rotation modulation the influence of temperature and magnetic-field-dependent viscosity is to destabilize the system as ferrofluids aid in advancing the onset of convection and thus enhancing the heat transport while the impact of rotation modulation is to stabilize the system.

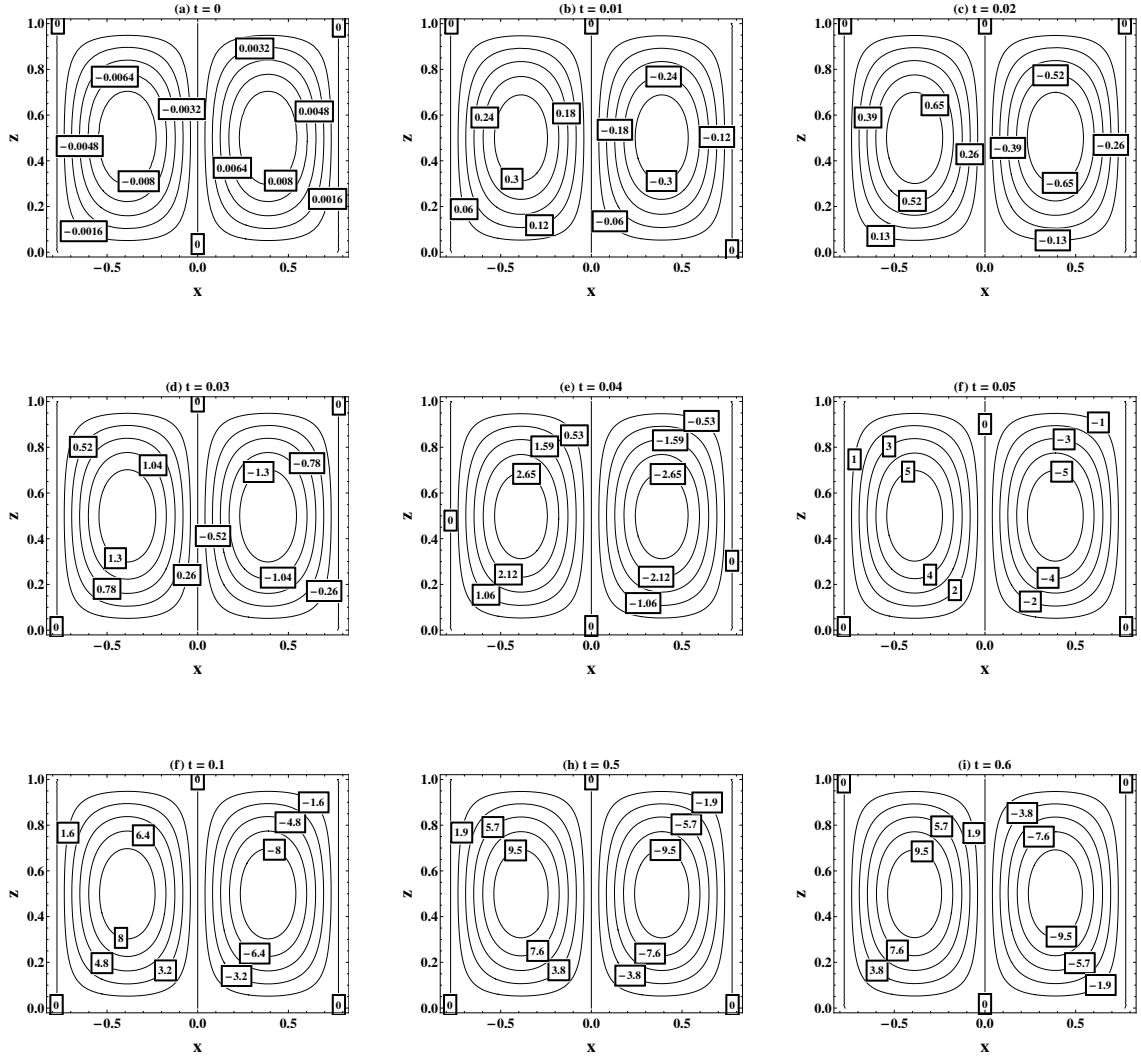


Figure 7.9: Streamlines for $Ta = 10, V = 0.3, M_1 = 1, M_3 = 1, \delta = 0.3, \omega = 20$.

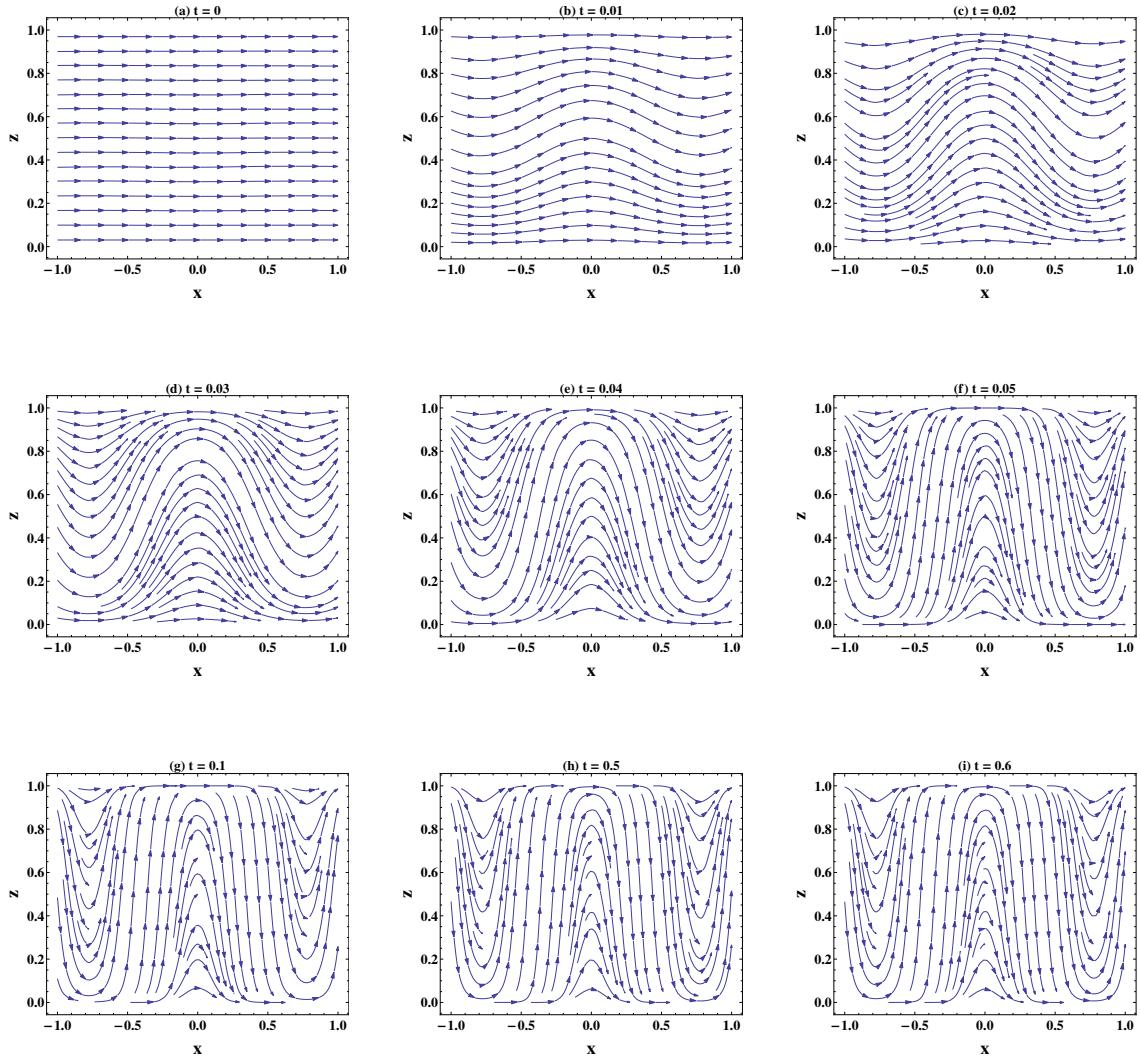


Figure 7.10: Isotherms for $Ta = 10, V = 0.3, M_1 = 1, M_3 = 1, \delta = 0.3, \omega = 20$ at different values of time, t.

7.3 Effects of Synchronous, Lower Wall and Asynchronous Boundary Temperature Modulations in a Ferrofluid with Variable Viscosity for Sinusoidal and Non-Sinusoidal Waveforms

We consider an infinite extent, incompressible Newtonian Boussinesq ferrrofluid with temperature and magnetic field dependent viscosity, $\mu(H,T)$ initially resting between two horizontal parallel upper and lower plates at $z=d$ and $z=0$ respectively which are kept at a finite depth, d. These parallel plates are maintained at different time periodic modulated temperatures with small amplitude, $\varepsilon^2\delta$, phase angle, ϕ and frequency of thermal modulation, ω. The externally imposed time and space dependent boundary conditions at the lower and upper boundaries [115] are given by

$$\left.\begin{aligned} T(0,t) &= T_0 + \frac{1}{2}\Delta T[1+\varepsilon^2\delta F_i(\omega,t)], \\ T(d,t) &= T_0 - \frac{1}{2}\Delta T[1+\varepsilon^2\delta F_i(\omega,t)]. \end{aligned}\right\} \tag{7.3.1}$$

where, $F_i(\omega,t)$ is the waveform of modulation with $i=1,2,3,4$ for the SqW, SW, TW and STW respectively. An uniform transverse magnetic field H_0 is applied in the vertical z-direction. The schematic of the problem is shown in Fig. 7.11.

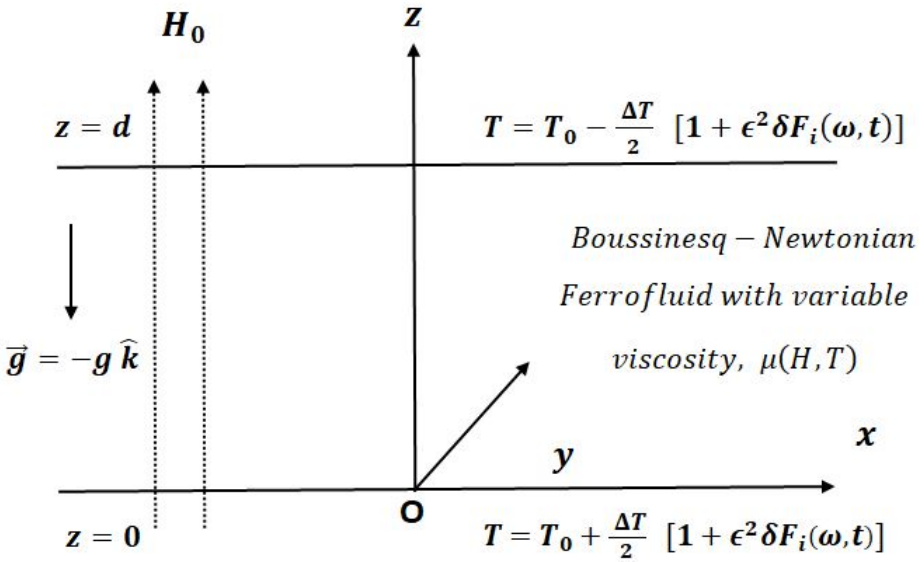


Figure 7.11: Schematic of Ferroconvection with boundary thermal modulation

The governing system of equations [80], [48] which describe this system of ferroconvection with thermorheological and magnetorheological effects are:

Conservation Equation of Mass

$$\nabla.\vec{q} = 0, \tag{7.3.2}$$

Conservation Equation of Linear Momentum

$$\rho_0\left[\frac{\partial\vec{q}}{\partial t}+(\vec{q}.\nabla)\vec{q}\right]=-\nabla p+\rho\vec{g}+\mu_0(\vec{M}.\nabla)\vec{H}+\nabla.\Big[\mu(H,T)(\nabla\vec{q}+\nabla\vec{q}^{Tr})\Big], \tag{7.3.3}$$

Conservation Equation of Energy

$$\rho_0 C_{VH}\Big[\frac{\partial T}{\partial t}+(\vec{q}.\nabla)T\Big]=\kappa\nabla^2 T, \tag{7.3.4}$$

Density Equation Of State

$$\rho=\rho_0[1-\alpha(T-T_0)], \tag{7.3.5}$$

Maxwell Equations

$$\nabla.\vec{B}=0 \qquad \nabla\times\vec{H}=0, \tag{7.3.6}$$

Magnetic Equation of State

$$M=M_0+\chi_m(\vec{H}-H_0)-K_1(T-T_0), \tag{7.3.7}$$

Effective Viscosity [53]

$$\mu(H,T)=\frac{\mu_0}{1+\delta_T(T-T_0)-\delta_H(\vec{H}-H_0)}. \tag{7.3.8}$$

The dimensionless equations which govern the disturbances when superposed over the basic state by using the scaling from and introducing the stream function, ψ in the form $u=-\dfrac{\partial\psi}{\partial z}, w=\dfrac{\partial\psi}{\partial x}$ and dropping the asterisks are:

$$\begin{aligned}\left[\frac{1}{Pr}\frac{\partial}{\partial t}(\nabla^2)-G_1(z)\nabla^4-2DG_1(z)\nabla^2\frac{\partial}{\partial z}-D^2G_1(z)\left(\frac{\partial^2}{\partial z^2}+\nabla_1^2\right)\right]\left(-\frac{\partial\psi}{\partial x}\right)=\\ RM_1D[f(z)]D(\nabla_1^2)\phi+R\left[\nabla_1^2-M_1D[f(z)]\nabla_1^2\right]T,\end{aligned} \tag{7.3.9}$$

$$\left(\frac{\partial}{\partial t}-\nabla^2\right)T=\frac{\partial\psi}{\partial x}\left[-1+\varepsilon^2\delta F_i(\omega,t)h_1(z)\right], \tag{7.3.10}$$

$$\left(M_3\nabla_1^2+\frac{\partial^2\phi}{\partial z^2}\right)\phi=\frac{\partial T}{\partial z}=\frac{\partial T}{\partial z}, \tag{7.3.11}$$

where

$$\left.\begin{aligned}&f(z)=T_b(z,t)-T_0, \qquad G_1(z)=\frac{1}{1+f(z)V},\\ &h_1(z)=Re\left[A(\lambda)e^{\lambda z}+A(-\lambda)e^{-\lambda z}\right]e^{-i\omega t}, \quad A(\lambda)=\frac{\Delta T}{2}\left[\frac{e^{-i\phi}-e^{-\lambda}}{e^{-\lambda}-e^{-\lambda}}\right], \quad \lambda=(1-i)\sqrt{\frac{\omega}{2}}.\end{aligned}\right\} \tag{7.3.12}$$

ψ, ϕ and T represent the non-dimensional perturbations for velocity, magnetic potential and temperature respectively.

The Eqns. (7.3.9) - (7.3.11) are solved subject to the stress-free, isothermal, and magnetic boundary conditions based on the assumption of infinite magnetic susceptibility with respect to the perturbed magnetic potential, in the form

$$\psi = \frac{\partial^2 \psi}{\partial z^2} = T = \frac{\partial \phi}{\partial z} = 0 \quad at \quad z = 0, 1. \tag{7.3.13}$$

We now obtain an equation in terms of ψ by combining Eqns. (7.3.9)-(7.3.11) in the form

$$\begin{aligned}\left(\frac{\partial}{\partial t} - \nabla^2\right)\left(M_3\nabla_1^2 + \frac{\partial^2}{\partial z^2}\right)\left[\frac{1}{Pr}\frac{\partial}{\partial t}\nabla^2) - G_1(z)\nabla^4 - 2DG_1(z)\nabla^2\frac{\partial}{\partial z} - D^2G_1(z)\left(\frac{\partial^2}{\partial z^2} + \nabla_1^2\right)\right]\left(-\frac{\partial \psi}{\partial x}\right) \\ = -\left(M_3\nabla_1^2 + \frac{\partial^2}{\partial z^2}\right)\left[R\left(\nabla_1^2 - M_1 D[f(z)]\right)\nabla_1^2\right]\left[-1 + \varepsilon^2\delta F_i(\omega, t)h_1(z)\right]\left(-\frac{\partial \psi}{\partial x}\right) \\ -RM_1\left[-1 + \varepsilon^2\delta F_i(\omega, t)h_1(z)\right]D[f(z)]D\nabla_1^2\frac{\partial}{\partial z}\left(-\frac{\partial \psi}{\partial x}\right)\end{aligned} \tag{7.3.14}$$

7.3.1 Linear Stability Analysis

We seek the solution of Eqn. (7.3.14) in the form:

$$\begin{aligned}\psi &= \psi_0 + \varepsilon\psi_1 + \varepsilon^2\psi_2 + \varepsilon^3\psi_3 + ... \\ R &= R_c + \varepsilon R_1 + \varepsilon^2 R_2 + \varepsilon^3 R_3 + ...\end{aligned} \tag{7.3.15}$$

where R_c is the Rayleigh number for the unmodulated ferroconvection problem. Following the Venezian [115] approach we arrive at the following:

$$R_c = \frac{k^2}{\pi^2\alpha^2}\frac{f(V)}{M}. \tag{7.3.16}$$

$$\begin{aligned}R_{2c} = \frac{R_c}{\pi^4\alpha^4(1+M_1)(M_3-\pi^2)}\sum_{n=1}^{\infty}|B_n(\lambda)|^2\left\{\pi^2(1-\pi^4\alpha^4) - M_3\pi^4\alpha^4 + \right. \\ \left.\pi^2\left(\frac{A_1^2 Y_1}{Y_1^2 + Y_2^2}\right)\left[1 + (\pi^4\alpha^4(M_1 - 1) - M_1)\right],\right.\end{aligned} \tag{7.3.17}$$

$$\begin{aligned}Y_1 = \pi^2 k_n(1+M_3\alpha^2)\left[\frac{-\omega^2\alpha^2}{Pr} + v_1 k_n^2 - 2v_2\pi k_n\right] - v_3\pi^2 k_n\left(1 - \alpha^2 + M_3\alpha^2(1-\alpha^2)\right) \\ -R_c\pi^4\alpha^2\left(1 + M_3\alpha^2(1+M_1)\right),\end{aligned} \tag{7.3.18}$$

$$Y_2 = \omega\pi^2 k_n(1+M_3\alpha^2)\left[\frac{-\alpha^2 k_n}{Pr} - v_1 k_n + 2v_2\pi\right] + v_3\omega\pi^4 k_n\left(1 - M_3\alpha^2(1+\alpha^2)\right), \tag{7.3.19}$$

We evaluate R_{2c} for the following three modulations as follows:

Case (i) : Synchronous Modulation (Inphase)

The wall temperatures are modulated inphase with $\Phi = 0$, i.e. the oscillating field is symmetric. In this case $B_n(\lambda) = b_n$ or 0, accordingly as n is even or odd.

Case (ii) : Lower Wall Modulation

The upper plate is maintained at a constant temperature and the bottom wall is modulated with $\Phi - i\infty$. In this case $B_n(\lambda) = \frac{1}{2} b_n$ or 0, accordingly as n is even or odd for integer values of n.

Case (iii) : Asynchronous Modulation (Out of phase)

The wall temperatures are modulated out of phase with $\Phi = \pi$, i.e. the oscillating field is antisymmetric. In this case $B_n(\lambda) = b_n$ or 0, accordingly as n is even or odd, where

$$b_n = \frac{-4n\pi^2\lambda^2}{[\lambda^2 + (n+1)^2\pi^2][\lambda^2 + (n-1)^2\pi^2]} \tag{7.3.20}$$

We note that in Eqn. (7.3.17) the summation extends over even values of n for case (i), odd values of n for case (ii) and all integer values for case (iii).

7.3.2 Non-Linear Stability Analysis

The dimensionless non-linear system of equations which govern ferroconvection with thermorheological and magnetorheological effect are:

$$\frac{1}{Pr}\frac{\partial}{\partial t}(\nabla^2\psi) = G_1(z)(\nabla^4\psi) + 2DG_1(z)(\nabla^2\psi) - D^2G_1(z)\left(\frac{\partial^2\psi}{\partial x^2} - \frac{\partial^2\psi}{\partial z^2}\right) - R\frac{\partial T}{\partial x}(1 - M_1) - RM_1\left(\frac{\partial^2\phi}{\partial x \partial z}\right) + RM_1 J\left(T, \frac{\partial\phi}{\partial z}\right) + \frac{1}{Pr}J(\psi, \nabla^2\psi), \tag{7.3.21}$$

$$\frac{\partial T}{\partial t} = \frac{\partial\psi}{\partial x}\left[-1 + \varepsilon^2\delta F_i(\omega, t)h_1(z)\right] + \nabla^2 T + J(\psi, T), \tag{7.3.22}$$

$$M_3\frac{\partial^2\phi}{\partial x^2} + \frac{\partial^2\phi}{\partial z^2} = \frac{\partial T}{\partial z}, \tag{7.3.23}$$

We assume the solution to the stream function, temperature and magnetic potential as the following minimal mode truncation form:

$$\psi = A_1(t)sin(\pi\alpha x)sin(\pi z), \tag{7.3.24}$$

$$T = A_2(t)cos(\pi\alpha x)sin(\pi z) + A_3(t)sin(2\pi z), \tag{7.3.25}$$

$$\phi = A_4(t)cos(\pi\alpha x)cos(\pi z) + A_5(t)cos(2\pi z), \tag{7.3.26}$$

On simplification we obtain the following set of ordinary differential equations for the time evolution of the amplitudes, A_i's which involve the properties of ferrofluids with variable viscosity and the influence of thermal modulation for different waveforms:

$$\begin{aligned}\dot{A_1}(t) = -\frac{Pr}{k^2}f(V)A_1(t) - \left[\frac{\pi\alpha PrR(1-M_1)}{k^2} + \frac{\pi\alpha PrRM_1}{k^2(1+M_3\alpha^2)}\right]A_2(t) \\ + \frac{\pi^2\alpha RPrM_1}{k^2}\left(\frac{1}{1+M_3\alpha^2} - 1\right)A_2(t)A_3(t),\end{aligned} \tag{7.3.27}$$

$$\dot{A_2}(t) = \pi\alpha\left[-1 + \varepsilon^2\delta F_i(\omega,t)h_1\right]A_1(t) - k^2A_2(t) - \pi^2\alpha A_1(t)A_3(t), \tag{7.3.28}$$

$$\dot{A_3}(t) = -4\pi^2A_3(t) + \frac{\pi^2\alpha}{2}A_1(t)A_2(t), \tag{7.3.29}$$

To study the amount of heat transport in the system, we determine the average Nusselt number using the following integral,

$$\overline{Nu(t)} = \frac{\omega}{2\pi}\int_0^{\frac{2\pi}{\omega}} Nu(t)dt. \tag{7.3.30}$$

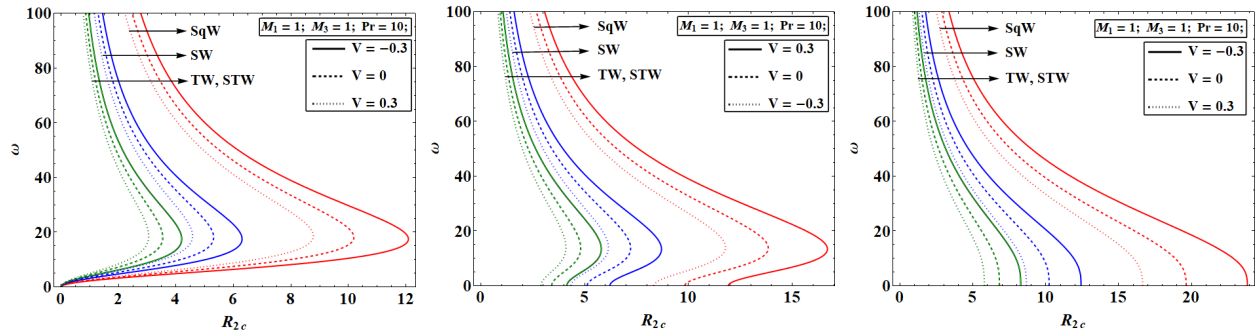
where,

$$Nu(t) = 1 - 2\pi A_3(t). \tag{7.3.31}$$

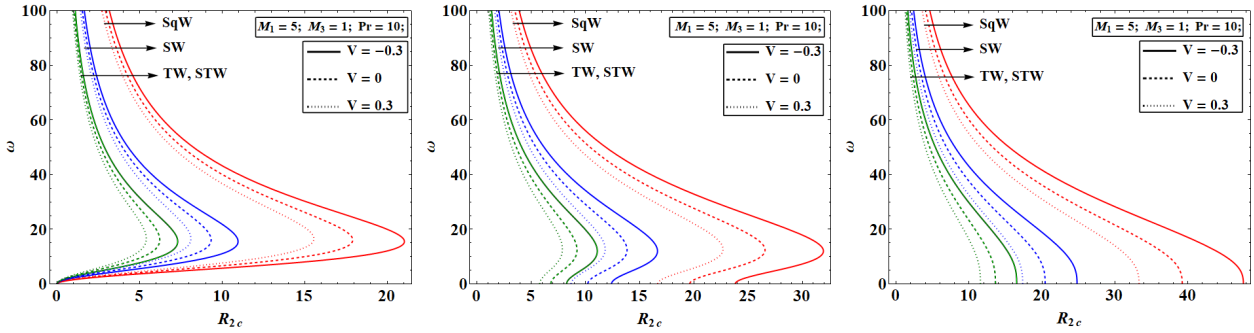
and $A_3(t)$ is calculated by solving the magnetic-Lorenz model in Eqns. (7.3.27)-(7.3.29).

Parameter	Effect of $\overline{Nu(t)}$ for different waveforms		
	Synchronous Modulation	Lower Wall Modualtion	Asynchronous Modulation
V	$\overline{Nu(t)}_{SqW} = \overline{Nu(t)}_{SW} = \overline{Nu(t)}_{TW} = \overline{Nu(t)}_{STW}$	$\overline{Nu(t)}_{SqW} > \overline{Nu(t)}_{TW} > \overline{Nu(t)}_{STW} > \overline{Nu(t)}_{SW}$	$\overline{Nu(t)}_{SqW} > \overline{Nu(t)}_{TW} > \overline{Nu(t)}_{STW} > \overline{Nu(t)}_{SW}$
M_1	$\overline{Nu(t)}_{SqW} = \overline{Nu(t)}_{SW} = \overline{Nu(t)}_{TW} = \overline{Nu(t)}_{STW}$	$\overline{Nu(t)}_{SqW} > \overline{Nu(t)}_{TW} > \overline{Nu(t)}_{STW} > \overline{Nu(t)}_{SW}$	$\overline{Nu(t)}_{SqW} > \overline{Nu(t)}_{TW} > \overline{Nu(t)}_{STW} > \overline{Nu(t)}_{SW}$
M_3	$\overline{Nu(t)}_{SqW} = \overline{Nu(t)}_{SW} = \overline{Nu(t)}_{TW} = \overline{Nu(t)}_{STW}$	For $M_3 < 40$: $\overline{Nu(t)}_{SqW} > \overline{Nu(t)}_{TW} > \overline{Nu(t)}_{STW} > \overline{Nu(t)}_{SW}$ For $M_3 \geq 40$: $\overline{Nu(t)}_{SqW} > \overline{Nu(t)}_{SW} > \overline{Nu(t)}_{TW} > \overline{Nu(t)}_{STW}$	For $M_3 < 40$: $\overline{Nu(t)}_{SqW} > \overline{Nu(t)}_{TW} > \overline{Nu(t)}_{STW} > \overline{Nu(t)}_{SW}$ For $M_3 \geq 40$: $\overline{Nu(t)}_{SqW} > \overline{Nu(t)}_{SW} > \overline{Nu(t)}_{TW} > \overline{Nu(t)}_{STW}$
Pr	$\overline{Nu(t)}_{SqW} = \overline{Nu(t)}_{SW} = \overline{Nu(t)}_{TW} = \overline{Nu(t)}_{STW}$	$\overline{Nu(t)}_{SqW} > \overline{Nu(t)}_{TW} > \overline{Nu(t)}_{STW} > \overline{Nu(t)}_{SW}$	$\overline{Nu(t)}_{SqW} > \overline{Nu(t)}_{TW} > \overline{Nu(t)}_{STW} > \overline{Nu(t)}_{SW}$

Table 7.6: Effect of thermal modulation for sinusoidal and non-sinusoidal waveforms on heat transport.

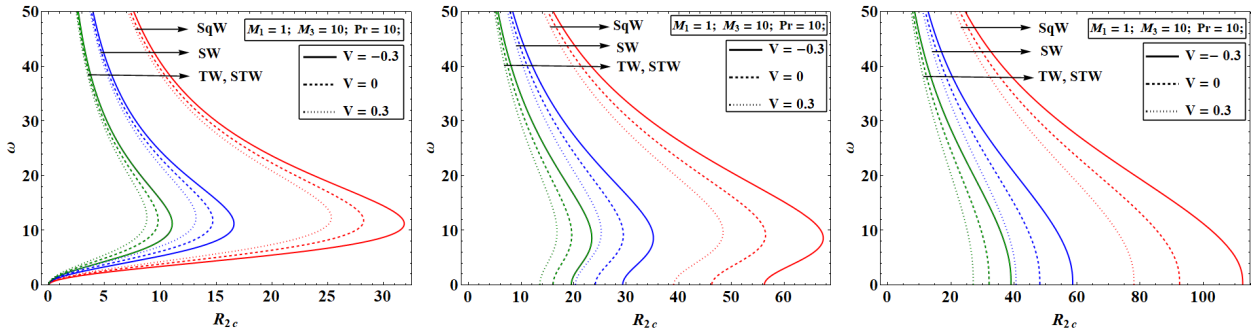


a: Synchronous Modulation b: Lower Wall Modulation c: Asynchronous Modulation

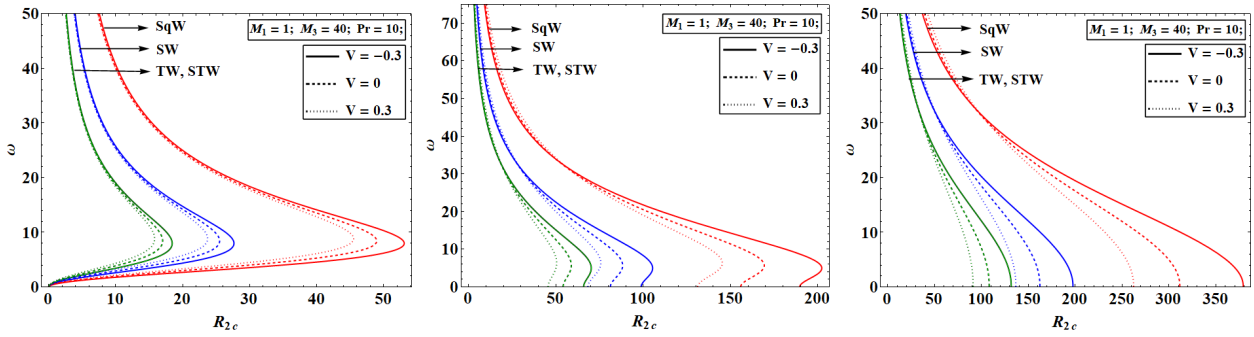


d: Synchronous Modulation e: Lower Wall Modulation f: Asynchronous Modulation

Figure 7.12: Plot of correction Rayleigh number, R_{2c}, vs. the frequency of modulation, ω, by varying the buoyancy magnetization parameter, M_1, for different values of the variable viscosity parameter, V, for different waveforms and modulations.

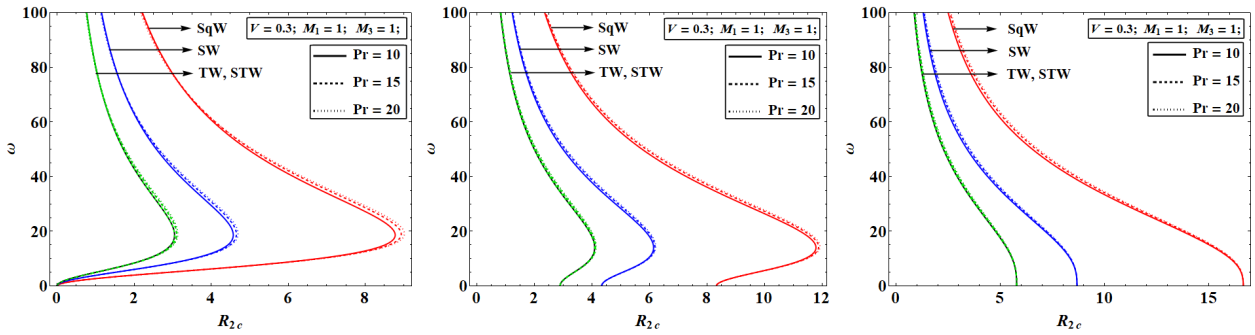


a: Synchronous Modulation b: Lower Wall Modulation c: Asynchronous Modulation



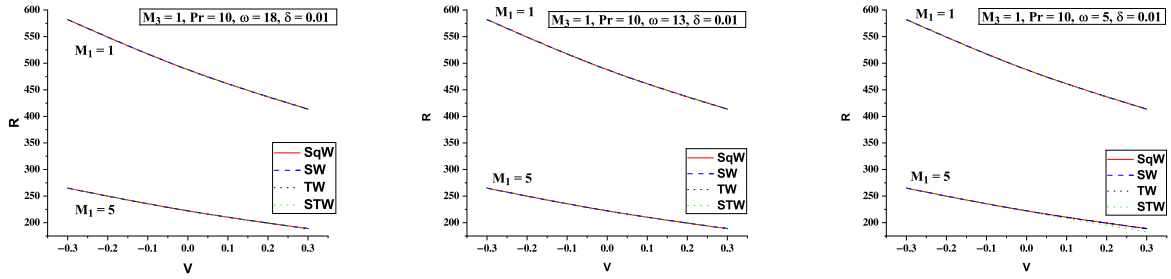
d: Synchronous Modulation e: Lower Wall Modulation f: Asynchronous Modulation

Figure 7.13: Plot of R_{2c} vs. ω, by varying the non-buoyancy magnetization parameter, M_3, for different values of V, for different waveforms and modulations.



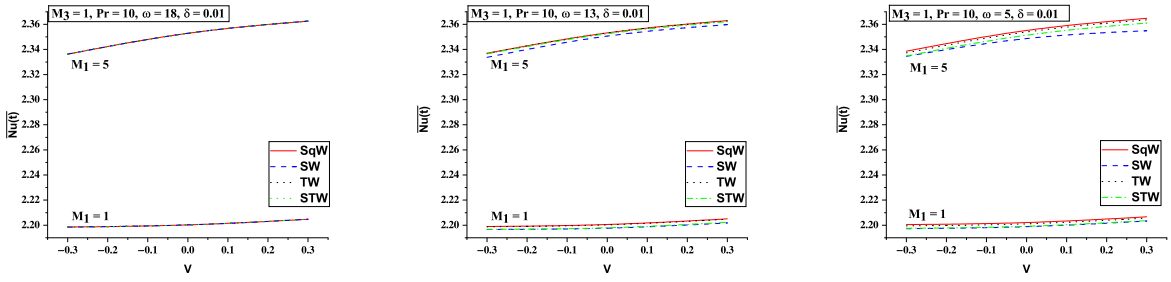
a: Synchronous Modulation b: Lower Wall Modulation c: Asynchronous Modulation

Figure 7.14: Plot of R_{2c} vs. ω, by varying the Prandtl number, Pr, for fixed values of the other parameters for different waveforms and modulations.



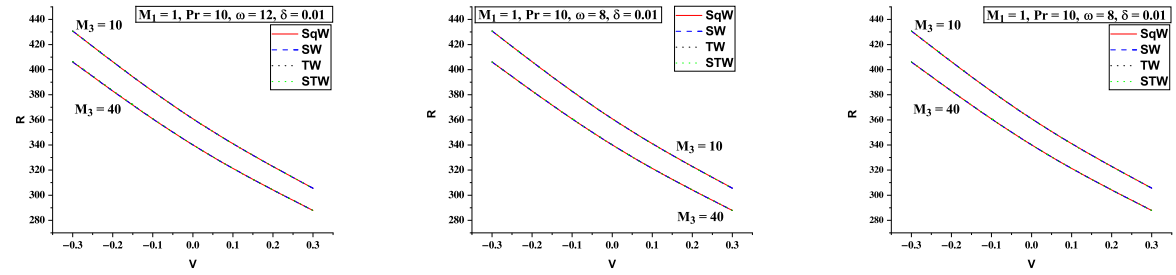
a: Synchronous Modulation b: Lower Wall Modulation c: Asynchronous Modulation

Figure 7.15: Plot of Rayleigh number, $R = R_c + \delta^2 R_{2c}$ vs. V for varying M_1.



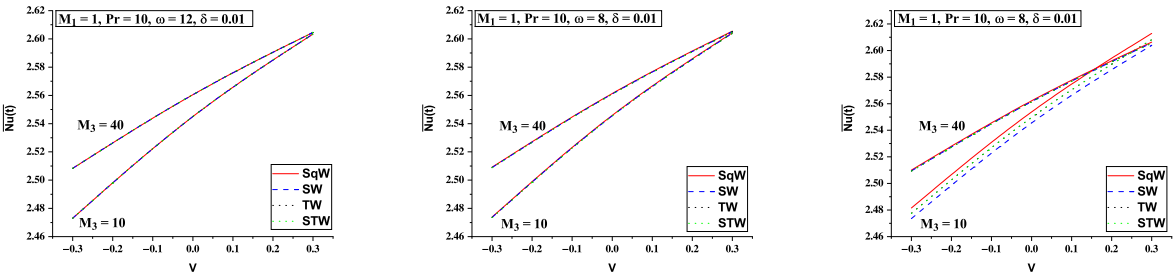
a: Synchronous Modulation b: Lower Wall Modulation c: Asynchronous Modulation

Figure 7.16: Plot of average Nusselt number, $\overline{Nu(t)}$ vs. V for varying M_1.



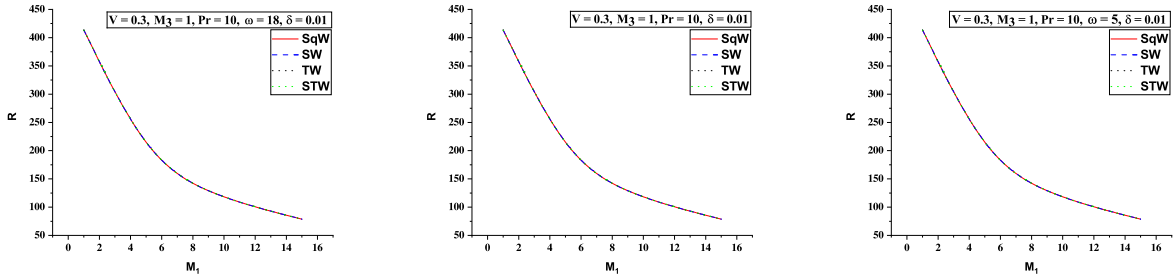
a: Synchronous Modulation b: Lower Wall Modulation c: Asynchronous Modulation

Figure 7.17: Plot of R vs. V for varying M_3.



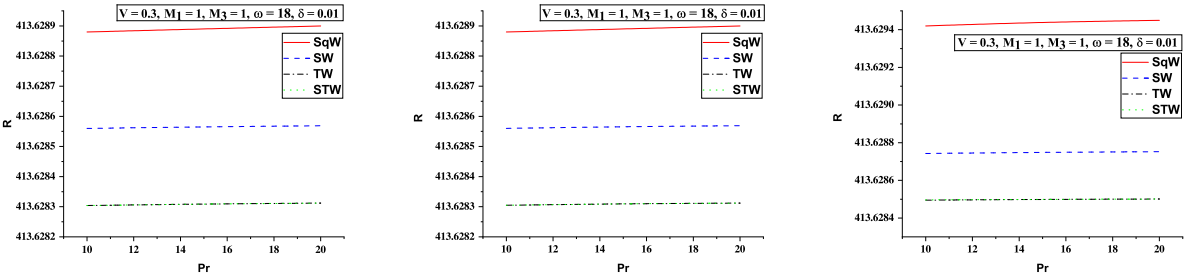
a: Synchronous Modulation b: Lower Wall Modulation c: Asynchronous Modulation

Figure 7.18: Plot of $\overline{Nu(t)}$ vs. V for varying M_3.



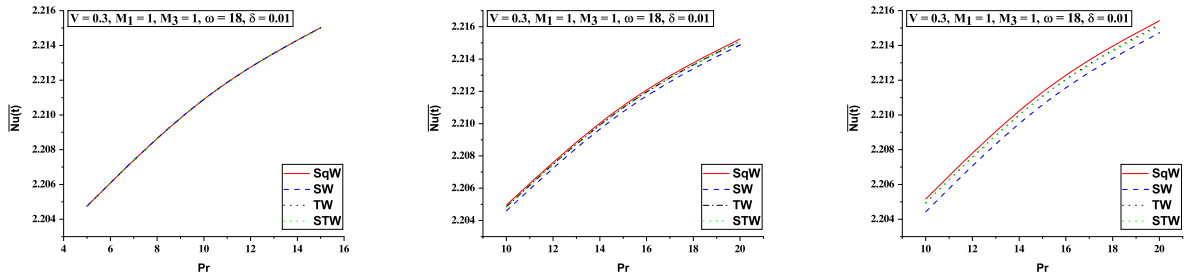
a: Synchronous Modulation b: Lower Wall Modulation c: Asynchronous Modulation

Figure 7.19: Plot of R vs. M_1 for fixed values of other parameters.



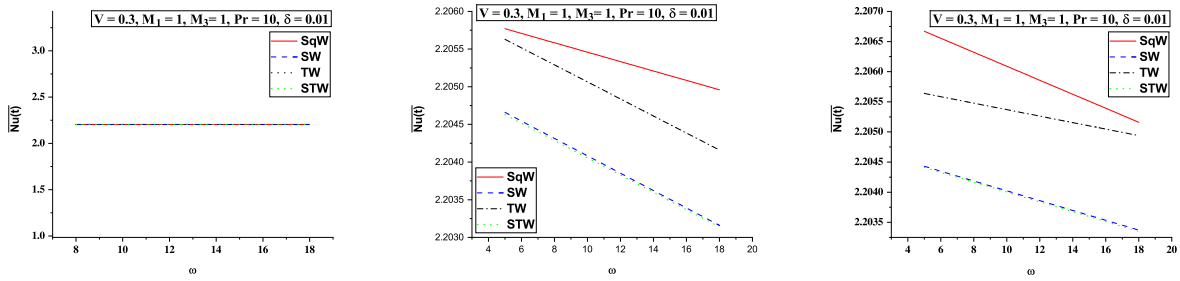
a: Synchronous Modulation b: Lower Wall Modulation c: Asynchronous Modulation

Figure 7.20: Plot of R vs. Pr for fixed values of other parameters.



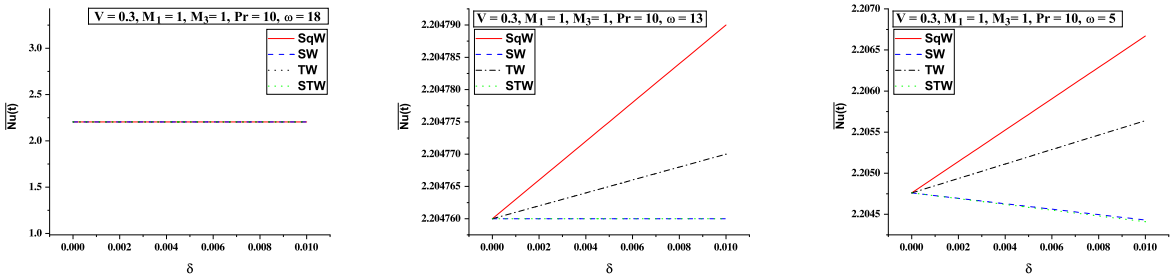
a: Synchronous Modulation b: Lower Wall Modulation c: Asynchronous Modulation

Figure 7.21: Plot of $\overline{Nu(t)}$ vs. Pr for fixed values of other parameters.



a: Synchronous Modulation b: Lower Wall Modulation c: Asynchronous Modulation

Figure 7.22: Plot of $\overline{Nu(t)}$ vs. frequency of modulation, ω for fixed values of other parameters.



a: Synchronous Modulation b: Lower Wall Modulation c: Asynchronous Modulation

Figure 7.23: Plot of $\overline{Nu(t)}$ vs. amplitude of modulation, δ for fixed values of other parameters.

7.3.3 Results

This study investigates the effects of thermal modulation on the convective stability of ferrofluid with variable viscosity for sinusoidal and non-sinusoidal waveforms. The following general conclusions are drawn,

1. $[\overline{Nu(t)}]_{\delta=0} \approx \overline{Nu(t)}]_{\delta\neq 0}$ for synchronous modulation. This implies that synchronous modulation cannot be used to regulate heat transport in a system.

2. The effect of different parameters on R and $\overline{Nu(t)}$ is as follows

   - Effect of effective variable viscosity parameter is given as

   $$[R_c]_{V<0} > [R_c]_{V=0} > [R_c]_{V>0}$$

   $$[\overline{Nu(t)}]_{V<0} < [\overline{Nu(t)}]_{V=0} < [\overline{Nu(t)}]_{V>0}$$

   Thus, in a ferrofluid the thermorheological effect aids a maximum heat transport while the magnetorheological effect results in a diminished transport of heat.

   - The magnetic numbers M_1 and M_3 hasten the heat transport in the system.
   - The effect of thermal modulation on R and $\overline{Nu(t)}$ can be given as:

   $$R_{\Phi=0} < R_{\Phi\to -i\infty} < R_{\Phi=\pi}$$

   $$[\overline{Nu(t)}]_{\Phi=0} < [\overline{Nu(t)}]_{\Phi\to -i\infty} < [\overline{Nu(t)}]_{\Phi=\pi}$$

3. The asynchronous boundary temperature modulation is found to be most stable

4. The average Nusselt number increases with increase in amplitude of modulation but decreases with increase in frequency of modulation.

5. The non-sinusoidal waveforms are found to hasten the heat transport for all the parameters within a certain range with the square waveform facilitating the maximum heat transport in the system.

Explanation of the Problem	**Authors**	**Method Employed**	**Results obtained that coincide with the present study**
With thermal modulation and for a non-magnetic fluid with constant viscosity	C Kanchana et al. [42]	Method of multiscales	• No influence of synchronous modulation • $\overline{Nu(t)}$ decreases with increase in ω (Fig. 7.22) • $[\overline{Nu(t)}]_{\Phi=0}$<$[\overline{Nu(t)}]_{\Phi\neq 0}$
Studies the effect of time-periodic temperature modualtion on thermal instability in a temperature dependent viscous fluid layer	Bhadauria [14]	Power series expansion	• Effect of synchronous modulation is negligible on heat transport. • Effect of increasing Pr is to advance the onset of convection in all the cases of modulation (Fig. 7.21) • Thermorheological parameter advances the heat transport.
Effect of modulation on onset of thermal convection Time periodic heating in RBC	Venezian [115] Bhadauria and Bhatia [13]	Perturbation expansion	On increasing the value of ω, the effect of thermal modulation on thermal instability disappears altogether. This confirms the results of [115, 13]

Table 7.7: Results from previous works and qualitative comparison with the present work.

Thus we conclude that in a ferrofluid with thermal modulation the influence of thermorheological and magnetorheological effects is to destabilize the system as ferrofluids aid in advancing the onset of convection and thus enhancing the heat transport. Among the three types of thermal modulations studied, asynchronous boundary temperature modulation is found to be most stable and the non-sinusoidal waveforms facilitate maximum heat transport.

7.4 Effect of Time-Periodic Magnetic Modulation on Rayleigh-Bénard Ferro-Convection with Variable Viscosity

Mathematical Formulation

The physical configuration of this problem is presented in Fig. which consists of horizontal plates of infinite thickness filled with a Boussinesq ferrofluid with variable viscosity, $\mu(H,T)$ heated from below. The origin of the cartesian co-ordinate system is assumed to be in the lower plate with z-axis directed vertically upwards and a uniform transverse magnetic field H_0 is applied in the vertical direction along the z axis. This system is under the influence of a time dependent sinusoidally and non-sinusoidaly varying magnetic field, $\vec{H}(t) = H_0[1+\varepsilon\delta^2 F_i(\omega,t)]$. Further a unifrom temperature gradient is maintained between the lower and upper boundaries. The boundaries are assumed to be stress free and isothermal.

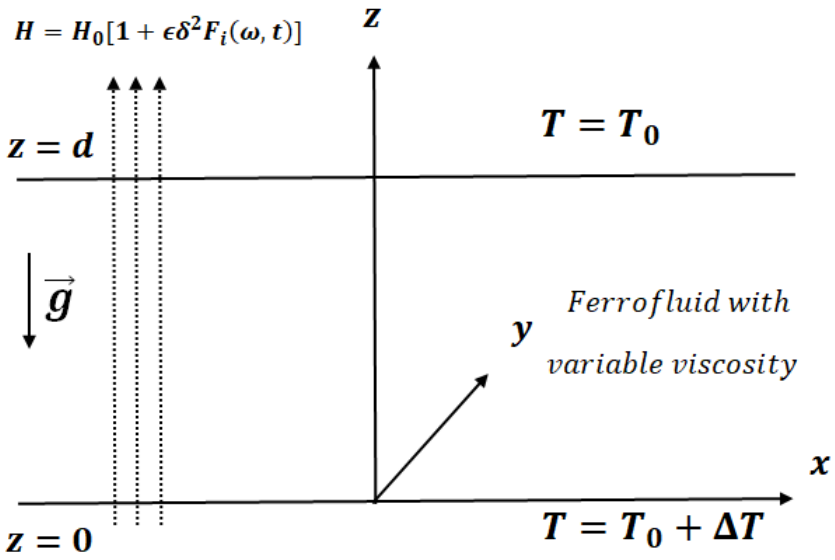


Figure 7.24: Schematic of Ferroconvection with magnetic modulation

The governing equations to study ferroconvection with variable viscosity under the influence of magnetic modulation are:

Conservation Equation of Mass

$$\nabla.\vec{q} = 0, \tag{7.4.1}$$

Conservation Equation of Linear Momentum

$$\rho_0 \left[\frac{\partial \vec{q}}{\partial t} + (\vec{q}.\nabla)\vec{q} \right] = -\nabla p + \rho \vec{g} + \mu_0 (\vec{M}.\nabla)\vec{H} + \nabla.\left[\mu(H,T)(\nabla \vec{q} + \nabla \vec{q}^{Tr}) \right] + \mu_m (\vec{H}.\nabla)\vec{H}), \tag{7.4.2}$$

Conservation Equation of Energy

$$\rho_0 C_{VH} \left[\frac{\partial T}{\partial t} + (\vec{q}.\nabla)T \right] = \kappa \nabla^2 T, \tag{7.4.3}$$

Density Equation Of State

$$\rho = \rho_0 [1 - \alpha(T - T_0)], \tag{7.4.4}$$

Magnetic Induction Equation

$$\frac{\partial \vec{H}}{\partial t} + (\vec{q}.\nabla)\vec{H} = \gamma_m \nabla^2 \vec{H} + (\vec{H}.\nabla)\vec{q} \tag{7.4.5}$$

Solenoidal Equation

$$\nabla.\vec{H} = 0 \tag{7.4.6}$$

Maxwell Equations

$$\nabla.\vec{B} = 0 \qquad \nabla \times \vec{H} = 0, \tag{7.4.7}$$

Magnetic Equation of State

$$M = M_0 + \chi_m(\vec{H} - H_0) - K_1(T - T_0), \tag{7.4.8}$$

Effective Viscosity [53]

$$\mu(H,T) = \frac{\mu_0}{1 + \delta_T(T - T_0) - \delta_H(\vec{H} - H_0)}. \tag{7.4.9}$$

The non-dimensional equations which govern the disturbances when superposed over the basic state take the following form:

$$\frac{1}{Pr}\frac{\partial}{\partial t}(\nabla^2\psi) = G_1(z)(\nabla^4\psi) + 2DG_1(z)(\nabla^2\psi) - D^2G_1(z)\left(\frac{\partial^2\psi}{\partial x^2} - \frac{\partial^2\psi}{\partial z^2}\right) - R\frac{\partial T}{\partial x}(1-M_1)$$
$$-RM_1\left(\frac{\partial^2\phi}{\partial x\partial z} - J\left(T, \frac{\partial\phi}{\partial z}\right)\right) + QP_m[1+\delta F_i(\omega,t)]\frac{\partial}{\partial z}\nabla^2\phi_H$$
$$-QP_mJ\left(\phi_H, \nabla^2\phi_H\right) + \frac{1}{Pr}J(\psi, \nabla^2\psi) \tag{7.4.10}$$

$$\left(\frac{\partial}{\partial t} - \nabla^2\right)T = -\frac{\partial\psi}{\partial x} + J(\psi, T), \tag{7.4.11}$$

$$\frac{\partial\phi}{\partial t} = P_m\nabla^2\phi + [1+\delta F_i(\omega,t)]\frac{\partial\psi}{\partial z} + J(\psi, \phi_H) \tag{7.4.12}$$

$$M_3\frac{\partial^2\phi}{\partial x^2} + \frac{\partial^2\phi}{\partial z^2} = \frac{\partial T}{\partial z}, \tag{7.4.13}$$

where

$$G_1(z) = \frac{1}{1+f(z)V}, \qquad f(z) = \left(1 - \frac{z}{d}\right).$$

The Eqns. (7.4.1)-(7.4.9) are solved subject to the stress-free, isothermal and magnetic boundary conditions based on the assumption of infinite magnetic susceptibility, in the from

$$\psi = \frac{\partial^2\psi}{\partial z^2} = \frac{\partial\phi_H}{\partial z} = T = \frac{\partial\phi}{\partial z} = 0 \quad at \quad z = 0, 1. \tag{7.4.14}$$

7.4.1 Non-Linear Stability Analysis

Using the idealized boundary conditions (7.4.14) a weakly non-linear stability analysis of the system (7.4.10)-(7.4.12) is performed by expanding the functions ψ, T, ϕ_H and ϕ as follows

$$\psi = A_1(t)sin(\pi\alpha x)sin(\pi z), \tag{7.4.15}$$

$$T = A_2(t)cos(\pi\alpha x)sin(\pi z) + A_3(t)sin(2\pi\alpha z), \tag{7.4.16}$$

$$\phi_H = A_4(t)sin(\pi\alpha x)cos(\pi z) + A_5(t)sin(2\pi\alpha x), \tag{7.4.17}$$

$$\phi = A_6(t)cos(\pi\alpha x)cos(\pi z) + A_7(t)cos(2\pi z), \tag{7.4.18}$$

The fifth order Lorenz model for a ferrofluid with variable viscosity under the effect of magnetic modulation is obtained as the following set of ordinary differential equations for the time evolution

of the amplitudes, A_i's:

$$\dot{A_1}(t) = -\frac{Pr}{k^2} f(V) A_1(t) - \frac{\pi\alpha PrRM}{k^2} A_2(t) - \frac{\pi^2 \alpha PrRM_1}{k^2} A_2(t)A_3(t) + \frac{2\pi\alpha PrRM_1}{k^2(1+M_3\alpha^2)} A_2(t)A_3(t) - \frac{QPrPm}{k^2}[1+\delta F_i(\omega,t)]\pi^3(1+\alpha^2)A_4(t) - \frac{\pi^4\alpha QPrPm}{k^2}(1-3\alpha^2)A_4(t)A_5(t) \tag{7.4.19}$$

$$\dot{A_2}(t) = -\pi\alpha A_1(t) - k^2 A_2(t) - \pi^2\alpha A_1(t)A_3(t), \tag{7.4.20}$$

$$\dot{A_3}(t) = -4\pi^2 A_3(t) + \frac{\pi^2\alpha}{2} A_1(t)A_2(t), \tag{7.4.21}$$

$$\dot{A_4}(t) = -k^2 Pm A_4(t) + [1+\delta F_i(\omega,t)]\pi A_1(t) + \pi^2\alpha A_1(t)A_5(t), \tag{7.4.22}$$

$$\dot{A_5}(t) = -4Pm\pi^2\alpha^2 A_5(t) - \frac{\pi^2\alpha}{2} A_1(t)A_4(t), \tag{7.4.23}$$

In the limit $V \to 0$ and $Q \to 0$, these equations reduce to the magnetic-Lorenz model of Laroze et al. [45] and in the limit $V \to 0$ reduces to the Lorenz model under the effect of magnetic modulation by Rudraiah [74] The slave amplitudes $A_6(t)$ and $A_7(t)$ are determined from the time independent equation for the scalar magnetic potential and are given as $A_6(t) = \dfrac{-A_2(t)}{\pi(1+M_3\alpha^2)}$ and $A_7(t) = -\dfrac{A_3(t)}{2\pi}$.

7.4.2 Linear Stability Analysis

Following the same technique of problem 7.1 we arrive at the following expressions to study the onset of ferroconvection:

Critical Rayleigh number, $\mathbf{R_c}$

$$R_c = \frac{k^2 f(V) + Q\pi^4(1+\alpha^2)}{\pi^2\alpha^2 M}. \tag{7.4.24}$$

Correction Rayleigh number, $\mathbf{R_{2c}}$

The correction Rayleigh number, R_{2c} takes the form,

$$R_{2c} = \frac{QPm^2\pi^2(1+\alpha^2)k^2\delta F_i(\omega,t)}{\alpha^2 M(k^4Pm^2+\omega^2)} - \frac{k^2}{\pi^2\alpha^2 PrM}\left\{N_1\left[(\omega^2+k^4Pm)Y_1 + ({}^2Pm - {}^{-2})Y_2\right] + N_2(k^2Y_1 - {}_2)\right\}, \tag{7.4.25}$$

$$Y_1 = \frac{\omega^2}{\pi\alpha} - \frac{Pr}{\pi\alpha} f(V) + \frac{\pi\alpha R_c PrM}{k^2} + \frac{QPrPm\pi^3(1+\alpha^2)}{\alpha}\left(\frac{\omega^2+k^4Pm}{\omega^2+k^4Pm^2}\right), \tag{7.4.26}$$

$$Y_2 = \frac{\omega k^2}{\pi\alpha} + \frac{Prf(V)\omega}{\pi\alpha k^2} - \frac{QPrPm\pi^3(1+\alpha^2)}{\alpha}\left(\frac{\omega k^2 Pm - \omega k^2}{\omega^2+k^4Pm^2}\right), \tag{7.4.27}$$

$$N_1 = \left(\frac{1}{\omega^2 + k^4 Pm^2}\right)\left(\frac{QPr\pi^4(1+\alpha^2)\delta^2 F_i(\omega,t)}{\alpha(Y_1^2+Y_2^2)}\right)\left(\frac{Pm^2k^2}{\omega^2+k^4Pm^2}+\frac{1}{k^4}\right) \tag{7.4.28}$$

$$N_2 = \left(\frac{QPr\pi^4(1+\alpha^2)\delta^2 F_i(\omega,t)}{\alpha(Y_1^2+Y_2^2)}\right)\left(\frac{Pm^2k^2}{\omega^2+k^4Pm^2}+\frac{1}{k^4}\right) \tag{7.4.29}$$

7.4.3 Results

The thermorheological and magnetorheological effects under time dependent magnetic field for sinusoidal and non-sinusoidal waveforms on Rayleigh-Bénard ferroconvection has been analyzed for both linear and non-linear analyses. The following conclusions are drawn:

1. The effect of time dependent magnetic field on RBF is to stabilize the system
2. The effect of the Chandrasekhar number, Q is to delay the onset of convection and hence decrease the heat transport.
3. The effect of increasing Pr and Pm is to advance the onset of convection and hence increase the heat transport.

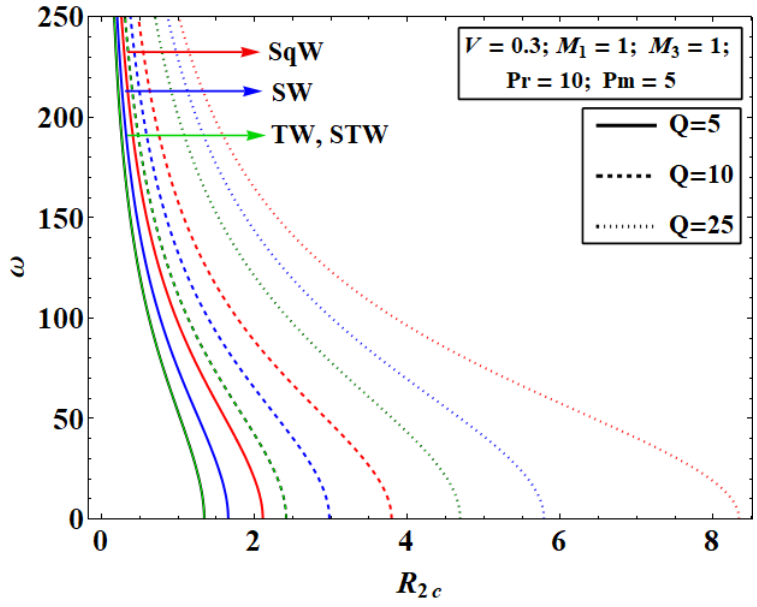
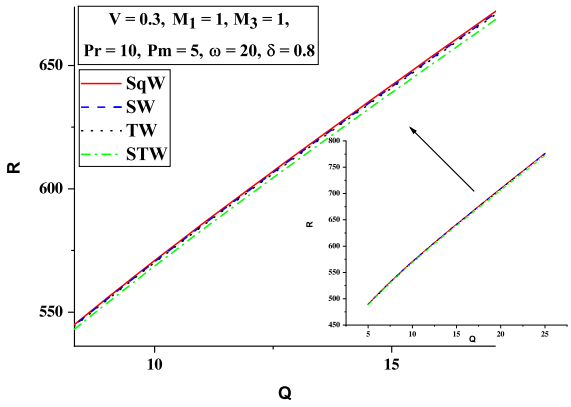
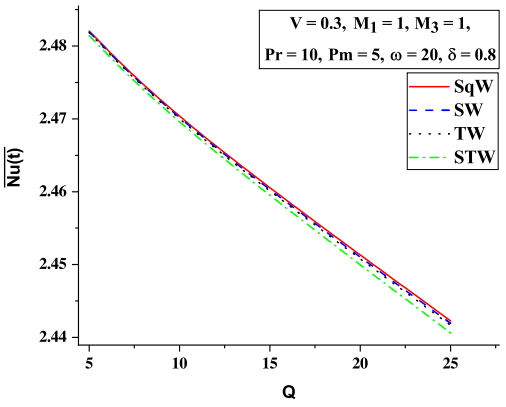


Figure 7.25: Plot of correction Rayleigh number, R_{2c} vs. the frequency of modulation, ω, for varying Chandrasekhar number, Q

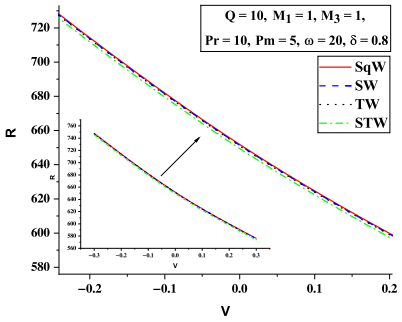


a: Plot of Rayleigh number, $R = R_c + \delta^2 R_{2c}$ vs. Q.

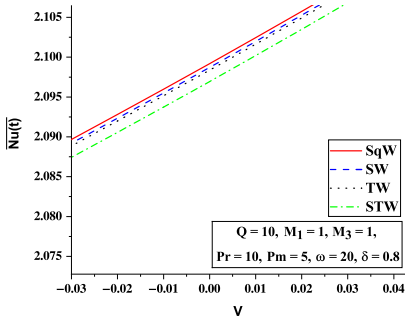


b: Plot of average Nusselt number, $\overline{Nu(t)}$ vs. Q

Figure 7.26: Plots of R and $\overline{Nu(t)}$ for varying Q.

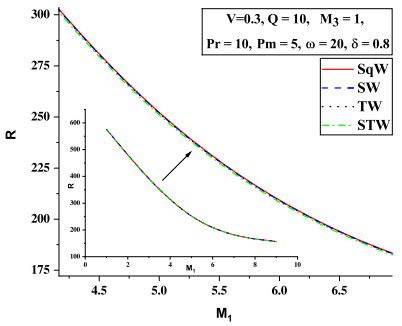


a: Plot of R vs. V.

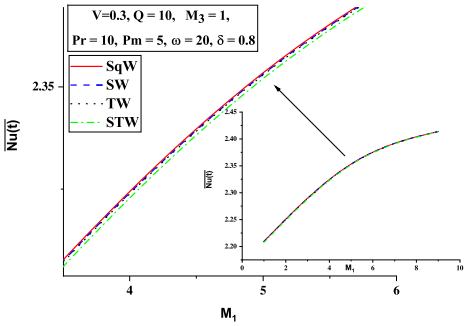


b: Plot of $\overline{Nu(t)}$ vs.V

Figure 7.27: Plots of R and $\overline{Nu(t)}$ for varying V.

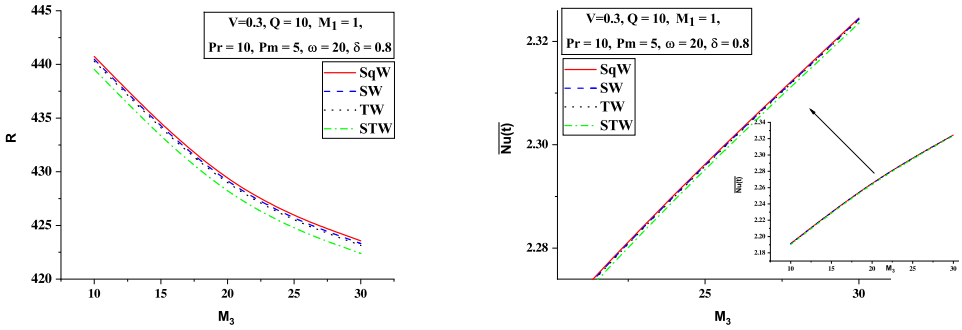


a: Plot of R vs. M_1.



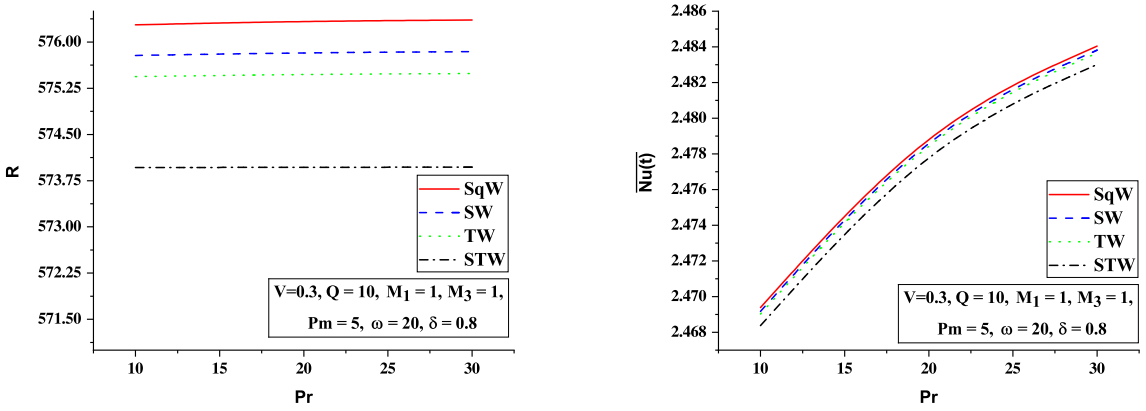
b: Plot of $\overline{Nu(t)}$ vs. M_1

Figure 7.28: Plots of R and $\overline{Nu(t)}$ for varying M_1.



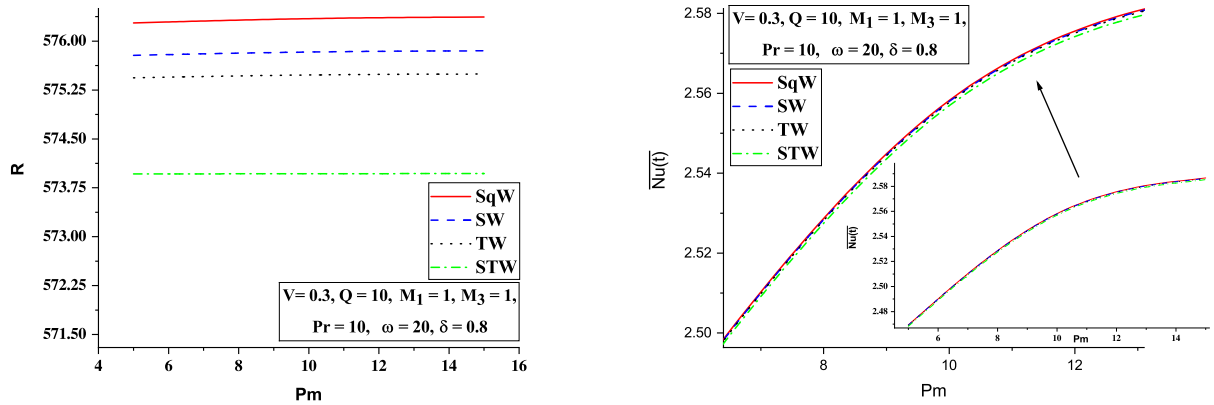
a: Plot of R vs. M_3. b: Plot of $\overline{Nu(t)}$ vs. M_3

Figure 7.29: Plots of R and $\overline{Nu(t)}$ for varying M_3.



a: Plot of R vs. Prandtl number, Pr. b: Plot of $\overline{Nu(t)}$ vs.Pr

Figure 7.30: Plots of R and $\overline{Nu(t)}$ for varying Pr.



a: Plot of R vs. magnetic Prandtl number, Pm. b: Plot of $\overline{Nu(t)}$ vs.Pm

Figure 7.31: Plots of R and $\overline{Nu(t)}$ for varying Pm.

Chapter 8

Presentations and Publications

Papers Presented:

Sl. No.	Title of the Article	Name of the Conference/Seminar/ Symposium	Institution	Year
1.	The effect of time periodic body force on heat transfer in a ferrofluid using Ginzburg-Landau Model.	International Conference on Fluid dynamics and its applications	BNM Institute of Technology, Bangalore	Jul. 2017
2.	Effect of time dependent Lorenz force in a ferrofluid	62nd International Conference of ISTAM	Osmania University, Hyderabad	Dec. 2017
3.	Effect of Modulations in Ferrofluids	National Conference on Emerging Trends in Mathematics and Applications in Engineering and Technology	Hindustan University, Chennai	Mar. 2018
4.	Convective Instability in a Vertically Oscillating Ferrofluid with Variable Viscosity	63rd International Conference of ISTAM	Dayananad Sagar University, Bangalore	Dec. 2018
5.	Effect of temperature-magnetic field dependent viscosity on ferroconvection with time-dependent body force	International Conference on Emerging Trends in Computational Fluid Dynamics, ICETCFD -2019	CHRIST (Deemed to be University), Bangalore	Feb. 2019
6.	Study of Thermorheological and Magnetorheological Effects in a Ferrofluid with Rotation Modulation	64th International Conference of ISTAM	IIT Bhubhaneshwar, Orissa	Dec. 2019
7.	Comparison of Four Different Waveforms of Temperature Modulation in a Ferrofluid with Variable Viscosity	International Conference on Emerging Trends in Fluid Mechanics and its Applications, ICFMA - 2020	CHRIST (Deemed to be University), Bangalore	Feb. 2020

Papers Published:

1. A. A. Neha, P. G. Siddheshwar, S. N. Nagouda and S. Pranesh, "Thermoconvective instability in a vertically oscillating horizontal ferrofluid layer with variable viscosity," *Heat Transfer*, Jun. 2020.

2. A. A. Neha, P. G. Siddheshwar, S. N. Nagouda and S. Pranesh, "Effects of Variable Viscosity and rotation Modulation on Ferroconvection," *J. Ther. Anal. Calorim.*, *(Communicated)-Under Review*

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